

## QUASI-MONTE CARLO METHODS AND PSEUDO-RANDOM NUMBERS

BY HARALD NIEDERREITER<sup>1</sup>

Nothing in Nature is random. . . . A thing appears random only through  
the incompleteness of our knowledge.

Spinoza, *Ethics* I

### CONTENTS

#### 1. Introduction

#### PART I. QUASI-MONTE CARLO METHODS

##### 2. Quasi-Monte Carlo integration

##### 3. Quasi-random points

##### 4. Good lattice points

##### 5. Application of diophantine approximations

#### PART II. PSEUDO-RANDOM NUMBERS

##### 6. Random numbers *vs.* pseudo-random numbers

##### 7. Linear congruential pseudo-random numbers

##### 8. Exponential sums

##### 9. Equidistribution test

##### 10. Interdependence of successive terms

##### 11. Serial test

**1. Introduction.** The subject matter of this talk is at the crossroads of two areas which will turn out to have more than only an etymological kinship, namely numerical analysis and number theory. Like so many mixed breeds, it has its fascinations and attractions, but also its inherent dilemmas. A multitude of concepts and devices dear to numerical analysts and computer users are, in open or disguised form, of an arithmetic nature, and problems arising in the computational workshop, especially those requiring effective methods, are now treated quite frequently with the powerful tools of the number theorist. This provides for a vivid interplay and is a source of enrichment for both disciplines. Of course, the occasion only permits us to look at a certain segment in the broad spectrum of activities. The leitmotif in our discussion will be the simulation of procedures containing an element of randomness by judiciously chosen deterministic schemes, with number theory playing a

---

This is an expanded version of an invited address presented at the 83rd Annual Meeting of the Society in St. Louis, Missouri, on January 27, 1977, under the title, *Quasi-Monte Carlo methods and pseudo-random numbers: Some applications of number theory*; received by the editors September 23, 1977.

AMS (MOS) subject classifications (1970). Primary 65-02, 65C05, 65C10, 65D30, 10F40, 10K05; Secondary 10-02, 10A35, 10F10, 10F20, 10G05, 10K30, 12A15, 65D05, 65N05, 65R05, 68A55.

<sup>1</sup> The author gratefully acknowledges support received from NSF grant MCS 77-01699.

© American Mathematical Society 1978