QUASI-MONTE CARLO METHODS AND PSEUDO-RANDOM NUMBERS

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Nothing in Nature is random. . . . A thing appears random only through the incompleteness of our knowledge.

Spinoza, Ethics I

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1. Introduction. The subject matter of this talk is at the crossroads of two areas which will turn out to have more than only an etymological kinship, namely numerical analysis and number theory. Like so many mixed breeds, it has its fascinations and attractions, but also its inherent dilemmas. A multitude of concepts and devices dear to numerical analysts and computer users are, in open or disguised form, of an arithmetic nature, and problems arising in the computational workshop, especially those requiring effective methods, are now treated quite frequently with the powerful tools of the number theorist. This provides for a vivid interplay and is a source of enrichment for both disciplines. Of course, the occasion only permits us to look at a certain segment in the broad spectrum of activities. The leitmotif in our discussion will be the simulation of procedures containing an element of randomness by judiciously chosen deterministic schemes, with number theory playing a

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