RAYS, WAVES AND ASYMPTOTICS[1]

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1. Introduction. In 1929 the American Mathematical Society established an annual lectureship named after Josiah Willard Gibbs (1839–1903), Professor of Mathematical Physics at Yale University from 1871 to 1903. Gibbs contributed essentially to the development of statistical mechanics and physical chemistry, and invented vector analysis. Therefore, it is appropriate that these lectures concern "mathematics or its applications" and "the contribution mathematics is making to present-day thinking and to modern civilization."

In this fiftieth Gibbs lecture, I will try to fulfill these objectives by describing some developments in the field of wave propagation. I hope that they will show also how mathematics itself is enriched by interaction with scientific and technical problems. In keeping with the intention that the lectures be "of a semipopular nature," I will omit as much technical detail as possible.

At first I was especially pleased that this is the fiftieth Gibbs lecture, because 50 is so special in our number system. This is because it is the product of the number of fingers on one hand multiplied by the number of fingers on two hands. But from this point of view, 50 is not a dimensionless number, since it has the dimensions of (fingers per hand) squared. Therefore, its numerical value depends upon the choice of units, so it has no intrinsic significance. This is a reminder that it is only dimensionless numbers which we can regard as large or small, as in the asymptotic analysis I am going to discuss later.

My plan is to begin with light rays and to describe their theory and use in optics. Then I will demonstrate some of their properties with the aid of a laser, kindly lent to me by Arthur Schawlow of Stanford University. Next, I will explain how rays were displaced by waves, which were introduced to provide a more accurate description of observed phenomena. The wave theory required solving certain partial differential equations, and numerous methods were devised to do this in special cases. However, in all other cases

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