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Harmonic analysis on compact solvmanifolds, by Jonathan Brezin, Lecture Notes in Math., vol. 602, Springer-Verlag, Berlin, Heidelberg, New York, 1977, v + 177 pp.

Harmonic analysis on nilpotent and solvable groups is something of a stepchild in today's mathematical family. Lacking the charisma of semisimple harmonic analysis and the relative docility of abelian harmonic analysis, it receives patronage neither from the élite nor from the masses. Though it enjoys occasional largesse from a variety of donors it must rely for its main livelihood on the benevolent researches of a small number of faithful sympathizers. Even that most famous of nilpotent groups, the cornerstone of the subject, an object of which every modern mathematician should be aware, the Heisenberg group, is rather far from being a household word. And the extremely elegant core of the theory, the method of orbits, probably first adumbrated by Harish-Chandra, but really exposed by A. A. Kirillov, and further substantially developed by Auslander-Kostant and others, probably still counts as specialist's knowledge.[1] This relative obscurity must be considered more a vagary of history than a divine judgement on the subject's intrinsic merits, for nilpotent and solvable harmonic analysis offers problems of depth and classical precedent. The small hardy band mentioned above has been patiently working on some of these. In recent years much of their attention has been directed towards problems of analysis on compact solvmanifolds, that is, on compact quotient spaces $\Gamma \backslash S$ where S is a solvable Lie group and Γ is a discrete subgroup. (If S is nilpotent, then $\Gamma \backslash S$ is called a nilmanifold.) The book under review offers a substantial survey of the work on solvmanifolds done to date. One of the book's strong points, one which makes it a good entryway for those curious about the subject, is the large amount of space devoted to representative examples.

If f is a function (C^∞, L^p, you choose–that's part of the fun) on $\Gamma \backslash S$, then f may be regarded as a function on S invariant under left translation by elements of Γ. That is, $f(\gamma s) = f(s)$ for γ in Γ and s in S. There is a naturally defined action ρ of S on the functions on $\Gamma \backslash S$ by right translation: $\rho(s')f(s) = f(ss')$ for s, s' in S. The basic problem of harmonic analysis in this context is, given a reasonable space of functions, to find the subspaces which are invariant under $\rho(S)$ and are minimal with respect to this property

[1]Kirillov's basic result on nilpotent groups can be stated so succinctly that I can't pass up this opportunity to disseminate it. Let N be a connected, simply connected nilpotent Lie group with Lie algebra $\mathfrak{N}$. Let Ad be the adjoint action of N on $\mathfrak{N}$. Let $\mathfrak{N}^*$ be the vector space dual to $\mathfrak{N}$ and Ad* the action of N contragredient to Ad. Then there is a canonical bijection between the set of equivalence classes of irreducible unitary representations of N and the collection of Ad*N orbits in $\mathfrak{N}^*$.