BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 4, July 1978

Decomposition of random variables and vectors, by Ju. V. Linnik and I. V. Ostrovskiĭ, Translations of Mathematical Monographs, Vol. 48, American Mathematical Society, Providence, R. I., 1977, ix + 380 pp., \$38.80. (Translated from the Russian, 1972, by Israel Program for Scientific Translations)

In the Foreword to his book of 1960, Linnik described decomposition of (probability) laws as "a field which in relation to mathematics employed lies between the theory of probability and the theory of functions of a complex variable", nowadays one would add "and of several complex variables." This field stands isolated from the mainstream of probability theory and is largely ignored. Thus it behooves us to be specific about its main concepts, problems, and representative results with their dates, in order to point out the evolution of the field before and after the crucial Linnik book which described its state as of 1960.

Until recently the central problem of probability theory was, and in large part still is, that of behaviour of sums of independent random variables. The inverse problem of decomposition of random variables, more precisely of their laws, was born–with its concepts, problems, and methods–during its heroic period 1934–1938 thanks to P. Lévy, Cramér, Hinčin, and Raikov. For twenty years it attracted little attention except mainly during 1947–1951 when Cramér, Lévy and Dugué produced various examples and counterexamples and Dugué introduced "ridge functions" which were to play an important role in factorizations of analytic characteristic functions. Thereafter Linnik's deep results, the impact of his book of 1960, and his personal influence attracted to the field a number of bright young mathematicians, especially Ostrovskiĭ–the joint author of the book under review. It presents the most exhaustive survey there is of the present state of the field.

The law $\mathfrak{L}$ of a random variable X is described by its distribution or by its distribution function F or by its characteristic function f, all with same affixes if any. The set of all laws is metrized by the Lévy metric $d(F_1, F_2)$. If $X = X_1 + X_2$ is sum of independent random variables X_1 and X_2, then its law $\mathfrak{L} = \mathfrak{L}_1\mathfrak{L}_2$ is described by the convolution, or composition, $F = F_1 * F_2$ or by the product $f = f_1 f_2$. $\mathfrak{L} = \mathfrak{L}_1\mathfrak{L}_2$ is "decomposable," or "factorizable," into "components" $\mathfrak{L}_1$ and $\mathfrak{L}_2$ if neither $\mathfrak{L}_1$ nor $\mathfrak{L}_2$ is degenerate. Decomposability is in fact if not in terminology a property of types of laws. P. Lévy (1937) produced "indecomposable," i.e. nondecomposable nondegenerate laws. On the other hand there are "infinitely divisible" or "infinitely decomposable" laws, "i.d." laws for short: $\mathfrak{L} = \mathfrak{L}_n^n$ for every integer $n > 0$. They were introduced by de Finetti (1929) and the characteristic functions of those with finite second moments were described explicitly by Kolmogorov (1932). The general form of i.d. characteristic functions was obtained by P. Lévy (1934) as a consequence of his exhaustive sample analysis of decomposable processes, i.e. of processes with independent increments. Hinčin (1937) gave a direct purely analytical proof of it: f is i.d. iff $f = e^{\psi}$ with $\psi = (\alpha, \Psi)$ given by