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BOOK REVIEWS

Nonlinear operators and differential equations in Banach spaces, by Robert H. Martin, Jr., Wiley, New York, 1976, xi + 440 pp., $27.50.

Ordinary differential equations in Banach spaces, by Klaus Deimling, Lecture Notes in Math., vol. 596, Springer-Verlag, Berlin, Heidelberg, New York, 1977, 136 pp.

In his celebrated lecture at the Paris Mathematical Congress of 1900, Hilbert asked whether the expansion of mathematical knowledge would not finally make it impossible for the single researcher to grasp all parts of it. As partial answer he said that all true progress goes hand in hand with the development of sharper aids and simpler methods, which make it easier to understand earlier theories and to circumvent complicated older procedures.

It would be hard to find better grounds for Hilbert's optimism than are given by current progress in the field of abstract differential equations. An excellent account of this current progress can be found in Martin's book (1976), also in *Ordinary differential equations in Banach spaces*, 1977, by Klaus Deimling. The latter is volume 596 of the Springer Lecture Notes, but is a much more scholarly and polished job than the category "lecture notes" would lead one to expect. Both Martin and Deimling are warmly recommended by this reviewer. They are complementary rather than competitive, and the pleasure of reading one is enhanced by reading the other.

A major difference in the two texts is that Martin includes background material which can be found in other books, while Deimling has chosen to emphasize current developments only. The former point of view has pedagogical advantages, but it reduces the scope below what one might hope for from the book's size and from the dust-jacket description. Thus, "the important, fundamental techniques of nonlinear functional analysis" could include the notion of topological degree, bifurcation theory, general nonlinear semigroups, and variational inequalities, all of which are, in fact, excluded. (This is said in no critical spirit, but simply as information.) Aside from a discussion of the connection between spectrum and numerical range in Chapter 3, and a development of fixed-point theory with application to Hammerstein equations in Chapter 4, the main strength of the book for research mathematicians would seem to be in the field of abstract differential equations. This is a field to which Martin (and also Deimling) have made fundamental contributions.

Let us begin with the differential equation $u' = f(t, u)$ where u is a function from an interval to a real Banach space X. When f is Lipschitzian, existence and uniqueness follow much as in the classical case dim $X < \infty$, in which case the equation is a finite system. By standard techniques (Dugundji,