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JORDAN ALGEBRAS AND THEIR APPLICATIONS

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In this article I want to sketch for nonexperts what Jordan algebras are why people might want to study such strange objects. I start with the assumption that the reader knows little or nothing about Jordan algebras, but has at least some respect for the terms Lie algebra, Lie group, Riemannian symmetric space, and bounded symmetric domain.

I. Jordan algebras in antiquity (1933–1966). I am unable to prove Jordan algebras were known to Archimedes, or that a complete theory has been found in the unpublished papers of Gauss. Their first appearance in recorded history seems to be in the early 1930's when the theory bursts forth full-grown from the mind, not of Zeus, but of Pascual Jordan, John von Neumann, and Eugene Wigner in their 1934 paper, *On an algebraic generalization of the quantum mechanical formalism* [**14**].

In the usual interpretation of quantum mechanics, the observables are Hermitian matrices (or Hermitian operators on Hilbert space) $x^* = x$, where the adjoint x^* is the conjugate transpose $\bar{x}^t$. The basic algebraic operations on such observables x are the matrix operations:

λx	multiplication by a complex scalar λ;
$x + y$	addition;
xy	matrix multiplication;
x^*	adjoint.

This formulation is open to the objection that the operations are not intrinsic to the physically significant part of the system: the scalar multiple λx is not again Hermitian unless the scalar is real, and the product xy is not again Hermitian unless x and y commute (or, as the physicists say, xy is not observable unless x and y are simultaneously observable). It was philosophically unsatisfactory to derive the algebraic structure from an unobservable operation xy, and, in addition, the matrix interpretation seemed insufficient when one attempted to apply quantum mechanics to relativistic and nuclear phenomena.

The program proposed by Jordan was to study the algebraic properties of Hermitian matrices without reference to the underlying (unobservable) matrix algebra. The strategy was:

(1) to formulate formal properties which seemed essential and physically significant;

(2) to consider abstract systems with these same formal properties taken as

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