BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 4, July 1978

CRYSTALLINE VARIATIONAL PROBLEMS

BY JEAN E. TAYLOR[1]

Surface tension is commonly thought of as a fluid phenomenon; the mere mention of the term brings to mind bugs skimming over water, liquids rising or falling in capillary tubes–and soap films and soap bubbles. But there is in fact a notion of surface tension (which is surface energy per unit surface area) for the interface between *any* two substances, or even between one substance and a vacuum. This surface energy arises from the fact that atoms (or molecules, or ions) of a given substance have a different environment at the interface between that substance and another than those in the bulk of the substance. (Sometimes even the composition of the surface is different from the bulk; this occurs for instance in soapy water having an interface with air.)

In this article we will deal with "surface tension functions" which are an outgrowth of the surface tensions of solids having their atoms arranged in some regular way. If one fixes the orientation of the lattice in R^3, then the environment of an atom on a planar interface between something else and such a regular structure can be different for different plane directions. Thus the surface tension between one substance and another can be a function

$$F: G_0(3, 2) \to R^+$$

where $G_0(3, 2)$ is the Grassmannian of oriented 2-planes through the origin in R^3 and R^+ is the set of positive real numbers. For the interface between, say, soapy water and air, the function F would be identically a constant; but for that between, say, a single crystal of ordinary salt (NaCl) and air, F would depend quite strongly on direction. If one wishes to allow the regular structure to vary over relatively long distances, one can even obtain a function

$$F: R^3 \times G_0(3, 2) \to R^+.$$

(This notion of surface tension for solid bodies is not at all esoteric, by the way: it is commonly used by metallurgists and others who deal with the surface properties of materials.)

Article based on an invited hour address to the Eastern Sectional Meeting of the American Mathematical Society in Wellesley, Massachusetts, October 22, 1977; received by the editors October 5, 1977.

AMS (MOS) subject classifications (1970). Primary 49F22, 49F20, 53C65, 49D99, 82A60, 82A65, 52A05, 52A40, 28A75, 49F10.

Key words and phrases. Calculus of variations, geometric measure theory, crystals, surface tension, approximation of minimal surfaces, prescribed boundary problem.

[1]This work was partially supported by NSF Grant MCS76-06424 and by a Sloan Fellowship.