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CONTINUOUS COHOMOLOGY OF GROUPS AND CLASSIFYING SPACES

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Topological groups exhibit one of the richest structures in mathematics, both because of the variety of significant examples and because of the interplay between topology and algebra. The examples range through geometry and into the study of differential equations. Indeed when Sophus Lie began to look at continuous groups around 1870 [35], he was particularly interested in those respecting a geometric structure and those respecting the solutions of a differential equation. By considering the solution sets of the equation as topological objects, it is possible to combine these two aspects; this is precisely a point of view now current in the theory of foliations. Following Lie, one finds interest not only in Lie groups but in Lie groupoids, for example, of *local* diffeomorphisms of a manifold.

To an algebraic topologist, there is a special challenge in the interplay between the algebra and topology of a topological group or groupoid. He can ignore the algebra and consider the cohomology of the underlying space or he can ignore the topology and consider the cohomology of the abstract group, but clearly neither is a satisfactory approach to these objects. To combine both the topology and the algebra, he has a variety of possibilities.

For any group G, let G^δ denote the corresponding group with the discrete topology. In particular, any abstract group can be thus topologized.

Looking at the cohomology of abstract groups as defined in terms of multivariable functions on the group, he can restrict attention to the continuous functions when dealing with a topological group G. This results in the continuous (group theoretic) cohomology $H_c(G)$.

Alternatively, since the abstract group cohomology $H(G) = H_c(G^\delta)$ is isomorphic with that of an associated space BG^δ, he can carry the topology into the construction of a corresponding classifying space BG and then consider $H(BG)$.

Finally for Lie groups, he can also consider the associated Lie algebra $\mathfrak{g}$ and the Lie algebraic cohomology $H(\mathfrak{g})$.

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