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# PSEUDOCONVEXITY AND THE PROBLEM OF LEVI

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The Levi problem is a very old problem in the theory of several complex variables and in its original form was solved long ago. However, over the years various extensions and generalizations of the Levi problem were proposed and investigated. Some of the more general forms of the Levi problem still remain unsolved. In the past few years there has been a lot of activity in this area. The purpose of this lecture is to give a survey of the developments in the theory of several complex variables which arise from the Levi problem. We will trace the developments from their historical roots and indicate the key ideas used in the proofs of these results wherever this can be done intelligibly without involving a lot of technical details. For the first couple of sections of this survey practically no knowledge of the theory of several complex variables is assumed on the part of the reader. However, as the survey progresses, an increasing amount of knowledge of the theory of several complex variables is assumed.

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## 1. Domains of holomorphy.

(1.1) One of the great differences between one complex variable and several complex variables is the concept of a domain of holomorphy. On any open subset $G$ of $\mathbf{C}$ there is a holomorphic function which cannot be extended across any boundary point of $G$. This is not the case in several complex variables, as was first pointed out by Hartogs [43]. The simplest example is the domain

$$\Omega = (\Delta_1 \times \Delta_{1/2}) \cup \left((\Delta_1 - \bar{\Delta}_{1/2}) \times \Delta_1\right)$$

where $\Delta_r$ is the open disc in $\mathbf{C}$ with center 0 and radius $r$. Any holomorphic

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