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TRANSFORMATIONS THAT DO NOT ACCEPT A FINITE INVARIANT MEASURE[1]

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In the following we shall consider only nonatomic, σ-finite measure spaces $(X, \mathfrak{B}, m)$. We say that a measure μ is equivalent to m if μ and m have the same sets of measure zero. We shall discuss measurable transformations T that are 1-1 onto maps with measurable inverses. If $m(TA) = m(A)$ for all sets $A \in \mathfrak{B}$ we say that T is a measure preserving transformation, or that m is an invariant measure for the transformation T. We assume that all transformations mentioned are nonsingular; in other words, the image of a set of positive measure has positive measure also. A measurable transformation T is ergodic if $TA = A$ implies that either the set A or its complement has measure zero. We shall often tacitly assume the phrase "almost everywhere", and all sets considered shall be measurable.

In [13] E. Hopf first discussed a necessary and sufficient condition for the existence of a finite invariant measure μ equivalent to m. Since then many authors have discussed different aspects of transformations without a finite invariant measure and have obtained a number of interesting results that have deep connections with other areas of mathematics. For instance, there is a significant influence on the classification theory of the factors of a von Neumann algebra.

In [10] weakly wandering sets for a transformation T were introduced, and it was shown that there exists a finite measure μ equivalent to m and invariant for T if and only if there are no weakly wandering sets of positive measure for the transformation T. Let $\mathfrak{W} = \{n_i | i = 0, 1, 2, \ldots\}$ be a sequence of integers; we say that a set A is a weakly wandering set under the sequence $\mathfrak{W}$ for the transformation T if the sets $T^{n_i}A$ for $i = 0, 1, 2, \ldots$ are mutually disjoint. If the weakly wandering set A has positive measure then we say that $\mathfrak{W}$ is a weakly wandering sequence for the transformation T. In case the sequence $\mathfrak{W}$ consists of the set of all integers then we have the familiar case where A is a wandering set.

Recurrent transformations play an important role in ergodic theory; these are the transformations which do not accept wandering sets of positive measure. Strongly recurrent transformations were introduced and discussed in [3]; these are the transformations which do not accept weakly wandering sets of positive measure. An ergodic measure preserving transformation T is

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