BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 2, March 1978

FACTORIZATION INDICES FOR MATRIX POLYNOMIALS

BY I. GOHBERG[1], L. LERER AND L. RODMAN

Communicated by R. G. Douglas, August 8, 1977

Let Γ be a rectifiable simple closed contour in the complex plane $\mathbf{C}$ bounding the domain F^+. The notation F^- will be used for the complement to $F^+ \cup \Gamma$ in $\mathbf{C} \cup \infty$. It will always be assumed that $\lambda = \infty \in F^-$.

A polynomial $L(\lambda) = A_0 + \lambda A_1 + \cdots + \lambda^m A_m$ with $n \times n$ matrix coefficients A_j and with $\det L(\lambda) \neq 0$ for $\lambda \in \Gamma$ is said to admit a (right standard) *factorization* relative to the contour Γ in case

$$L(\lambda) = L_+(\lambda)D(\lambda)L_-(\lambda), \tag{1}$$

where $L_+(\lambda)$ is a matrix polynomial with $\det L_+(\lambda) \neq 0$ for $\lambda \in F^+ \cup \Gamma$, $L_-(\lambda)$ is a matrix polynomial in the variable $(\lambda - a)^{-1}$ for some $a \in F^+$ and $\det L_-(\lambda) \neq 0$ for $\lambda \in F^-$, and

$$D(\lambda) = \operatorname{diag}((\lambda - a)^{\kappa_1}, (\lambda - a)^{\kappa_2}, \ldots, (\lambda - a)^{\kappa_n})$$

with some nonnegative integers $\kappa_1 \leqslant \kappa_2 \leqslant \cdots \leqslant \kappa_n$. Such a factorization of $L(\lambda)$ is not unique, but the numbers $\kappa_1 \leqslant \kappa_2 \leqslant \cdots \leqslant \kappa_n$ which are called the (*right*) *partial indices* are uniquely determined by $L(\lambda)$ (see [1], [5]). An analogous definition of left standard factorization is possible. Here we deal only with right factorization.

The factorization indices play an important role in the theory of systems of singular integral equations (see [1], [5]), Wiener-Hopf equations, partial differential equations, and the classification of holomorphic vector bundles on the Riemann sphere. There exists an algorithm for computing the indices which is described in [1].

In this paper we obtain some explicit formulas for the partial indices. As a main tool, we use the ideas of spectral analysis of matrix polynomials developed in [2], [3], [4].

To begin with, consider the linear case $L(\lambda) = A - \lambda I$. Let $A = \operatorname{diag}(A_1, A_2)$ be a block representation where the eigenvalues of A_1 are inside F^+ and the eigenvalues of A_2 are inside F^-. Then (assuming that $\lambda = 0$ is inside F^+)

AMS (MOS) subject classifications (1970). Primary 30A88, 45E05, 45E10; Secondary 15A18.

Key words and phrases. Factorization, partial indices, boundary value problem for analytic function, matrix valued polynomials.

[1] Partial supported by NSF grant at State University of New York at Stony Brook.

Copyright © 1978, American Mathematical Society