appear to some that years of herculean effort have yielded limited progress, but after all the antagonist is a formidable foe.

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The Selberg trace formula for PSL (2, **R**), Volume I, by Dennis A. Hejhal, Lecture Notes in Mathematics, Springer-Verlag, Berlin, Heidelberg, New York, 1976, iv + 516 pp., $ 15.20.

For the last twenty-five years or so the Selberg trace formula has had, in the general mathematical community, an aura of mystery which is only slowly dissipating. This circumstance makes it necessary for us to look a little at the history and nature of this formula in order to understand properly the position of this new book.

First of all, the Selberg trace formula has precedents some of which are very old indeed. The underlying technical ideas have been in common currency amongst applied mathematicians since the turn of the century; these arose in the study of Laplace's equation and we would now associate them with groups like $O(3, \mathbf{R})$. Furthermore, various versions are to be found in earlier investigations concerning automorphic forms. These were mostly number-theoretical and hinged around the class-number formulae discovered by Kronecker and studied further by Fricke, Mordell, Hecke and Eichler. But also from the differential-geometric point of view both J. Delsarte and H. Huber came very close to an explicit trace formula (for $PSL(2, \mathbf{R})$).

Yet, nevertheless, Selberg's discovery of this formula in the early 1950's was a revolutionary event and its impact is far from spent. This lies in the nature of the formula. Although I have continually referred to it as a *formula* it is much more a *method*; a method, that is, for probing more deeply into the nature of discontinuous groups and their function theory. In broad terms, the Selberg trace formula arises when one learns to think functional-analytically about automorphic functions and forms. This has been the *pons asinorum*; it forces one to shed preferences for complex-analytic functions and prejudices against 'soft analysis'. Once this has been done a new land, full of promise, opens up.

There are two approaches to the trace formula; that due to Selberg which uses differential and integral operators–and in fact the differential operators can be eliminated–and that due to Gelfand and his collaborators which uses representation theory. The latter is now almost indispensable for general, especially number-theoretic questions, whereas for the study of Fuchsian groups the former is more flexible. It is this that is used in this book and we shall first look at it a little more closely.

The basic idea is the following. Let S be a 'good' topological space and m a measure on S. Let A be a commutative family of compact integral operators on $L^2(S, m)$ and we suppose that the adjoint of any operator in A is also in A. Then, from spectral theory, we know that A can be 'diagonalised' and under our assumptions there exists a countable orthonormal basis $\{v_n;\ n \in \mathbf{N}\}$ of