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A FUBINI THEOREM FOR ITERATED STOCHASTIC INTEGRALS

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Let $x(t)$ and $y(t)$ be continuous martingales on the probability space $(\Omega, \mathcal{F}, \mathbf{P})$. Consider a bounded region $\mathcal{D}$ in $\mathbf{R}_+^2$ with smooth boundary Γ. Define Γ_i to be the portion of Γ along which the outward normal points in the direction of quadrant i $(i = 1, 2, 3, 4)$. Let $f(t, s)$ be a smooth function on $\overline{\mathcal{D}}$. It is desired to evaluate

$$\iint_{\mathcal{D}} f(t, s)\, dx(s) dy(t) \quad \text{and} \quad \iint_{\mathcal{D}} f(t, s)\, dy(t) dx(s)$$

by Riemann sums in an Ito-belated fashion. The difference between these sums is then of the form

$$\pm \Sigma f(t_i', s_i')[x(s_{i+1}') - x(s_i')]\,[y(t_{i+1}') - y(t_i')]$$

where (t_i', s_i') are points near Γ_2 and Γ_4. Under suitable conditions on $\mathcal{D}$ this sum tends to an integral of f along these portions of Γ. These considerations lead to the following

THEOREM (THE CORRECTION FORMULA). *Let $(x(t), y(t))$ be a joint martingale. Then*

$$\iint_{\mathcal{D}} f(t, s)\, dx(s) dy(t) + \int_{\Gamma_2 \cap l} f(t, t)\, d\langle x, y\rangle(t)$$
$$= \iint_{\mathcal{D}} f(t, s)\, dy(t) dx(s) + \int_{\Gamma_4 \cap l} f(t, t)\, d\langle x, y\rangle(t)$$

where $\langle x, y\rangle$ is the quadratic covariation process of x and y, and l is the line $s = t$.

It is to be noted that the rigorous justification of the Correction Formula entails the development of a new type of stochastic integral $I(t) = \int_{t_0}^t g(t, s)\, dx(s)$ where $g(t, s)$ is measurable with respect to the sigma-field generated by $\{x(u) - x(s): s \leqslant u \leqslant t\}$. Conditions are given which insure the existence of $I(t)$ as a limit of Ito-belated Riemann sums. The following result concerning the moments of $I(t)$ is presented.

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