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UPPER AND LOWER ESTIMATES ON THE RATE OF CONVERGENCE OF APPROXIMATIONS IN H_p

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Let $1 < p \leqslant \infty$ and let $H_p(U)$ denote the family of all functions f that are analytic in the unit disc U and such that

$$(1)\qquad \|f\|_p = \lim_{r\to 1}\left(\frac{1}{2\pi}\int_0^{2\pi}|f(re^{i\theta})|^p\,d\theta\right)^{1/p} < \infty.$$

Let σ_n be defined by

$$(2)\qquad \sigma_n = \inf_{w_j\in C, x_j\in U}\ \sup_{f\in H_p(U), \|f\|_p=1}\left|\int_{-1}^{1} f(x)\,dx - \sum_{j=1}^{n} w_j f(x_j)\right|.$$

We announce the following result.

THEOREM 1. *Given any $\epsilon > 0$, there exists an integer $n(\epsilon) \geqslant 0$ such that whenever $n > n(\epsilon)$, then*

$$(3)\qquad \exp[-(5^{1/2}\pi + \epsilon)n^{1/2}] \leqslant \sigma_n \leqslant \exp\left[-\left(\frac{\pi}{(2q)^{1/2}} - \epsilon\right)n^{1/2}\right],$$

where $q = p/(p-1)$.

Next, let $H_p^*(U)$ denote the family of all functions g such that $f \in H_p(U)$, where $f(z) = g(z)/(1 - z^2)$, and such that $H_p^*(U)$ is normed by $\|g\|_p^* = \|f\|_p$, where $\|f\|_p$ is defined as in (1). Let $g \in H_p^*(U)$, and let $\{T_n(g)\}$ be a linear approximation scheme defined by

$$(4)\qquad T_n(g)(z) = \sum_{j=1}^{n} g(x_j)\phi_{n,j}(z),\ xj \in U$$

where $\phi_{n,j}$ is analytic in U for each n and j, and such that

$$(5)\qquad \|T_n(g)\|_p^* \leqslant C\|g\|_p^*$$

where C is independent of n. We then announce

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