UPPER AND LOWER ESTIMATES ON THE RATE OF CONVERGENCE OF APPROXIMATIONS IN $H_p$

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Let $1 < p < \infty$ and let $H_p(U)$ denote the family of all functions $f$ that are analytic in the unit disc $U$ and such that

$$(1) \quad \|f\|_p = \lim_{r \to 1} \left( \frac{1}{2\pi} \int_{0}^{2\pi} |f(re^{i\theta})|^p \, d\theta \right)^{1/p} < \infty.$$ 

Let $\sigma_n$ be defined by

$$(2) \quad \sigma_n = \inf_{w_j \in C, x_j \in U} \sup_{f \in H_p(U), \|f\|_p = 1} \left| \int_{-1}^{1} f(x) \, dx - \sum_{j=1}^{n} w_j f(x_j) \right|.$$ 

We announce the following result.

\textbf{Theorem 1.} Given any $\epsilon > 0$, there exists an integer $n(\epsilon) \geq 0$ such that whenever $n > n(\epsilon)$, then

$$\exp\left(-\frac{1}{2} - \epsilon n^{1/2}\right) \leq \sigma_n \leq \exp\left(-\frac{\pi}{(2q)^{1/2}} - \epsilon\right) n^{1/2},$$

where $q = p/(p - 1)$.

Next, let $H^*_p(U)$ denote the family of all functions $g$ such that $f \in H_p(U)$, where $f(z) = g(z)/(1 - z^2)$, and such that $H^*_p(U)$ is normed by $\|g\|_p^* = \|f\|_p$, where $\|f\|_p$ is defined as in (1). Let $g \in H^*_p(U)$, and let $\{T_n(g)\}$ be a linear approximation scheme defined by

$$(3) \quad T_n(g)(z) = \sum_{j=1}^{n} g(x_j) \phi_{n,j}(z), \quad x_j \in U$$

where $\phi_{n,j}$ is analytic in $U$ for each $n$ and $j$, and such that

$$\|T_n(g)\|_p^* \leq C\|g\|_p^*$$

where $C$ is independent of $n$. We then announce

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