BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 84, Number 1, January 1978

AVERAGE GAUSSIAN CURVATURE OF LEAVES OF FOLIATIONS

BY DANIEL ASIMOV[1]

Communicated by P. T. Church, July 26, 1977

Let F be a smooth transversely-oriented foliation of a compact, connected, oriented, Riemannian manifold W^{n+1} of constant sectional curvature $\equiv c$. Let K_F: $W \longrightarrow \mathbf{R}$ via $K_F(x)$ = the Gaussian curvature (defined below) of the leaf l^n through x at x. For $n = 2$ this is classical Gaussian curvature. Let vol be the canonical volume on W, and define $\overline{K}_F$ by Volume $(W) \cdot \overline{K}_F = \int_W K_F \,\mathrm{vol}$.

THEOREM 1.

$$\overline{K}_F = \begin{cases} 2^n c^{n/2} \Big/ \binom{n}{n/2}, & n \text{ even}, \\ 0, & n \text{ odd}. \end{cases}$$

THEOREM 2. *Let $n + 1 = 3$ and suppose F, W, c are as above except that ∂W is nonempty and is a union of leaves of F. Then*

$$\int_W K_F \mathrm{vol} = 2c \,\mathrm{Volume}(W) + \int_{\partial W} H \,\mathrm{vol}'$$

where H: $\partial W \longrightarrow \mathbf{R}$ is the mean curvature (computed with respect to the transverse orientation), and vol′ *is the canonical volume on ∂W.*

THEOREM 3. *Suppose $n + 1 = 3$. Let F and W be as in the original hypotheses with $\partial W = \varnothing$ but assume the sectional curvatures of W lie between c_1 and c_2. Then we have $2c_1 \leqslant \overline{K}_F \leqslant 2c_2$.*

DEFINITION OF GAUSSIAN CURVATURE. We define, for a Riemannian manifold $l = l^n$, the function K: $l \longrightarrow \mathbf{R}$ in two cases (which overlap):

Case (i). n is even. In this case a local orthonormal frame on l gives rise to a matrix of curvature 2-forms, $\Omega = (\Omega^i_j)$ defined locally. The Pfaffians of the local Ω agree on overlaps and so define a global n-form $\mathrm{Pf}(\Omega)$ on l. Letting ν denote the canonical volume form on l we set

$$K\nu = \frac{2^{n/2} \cdot (n/2)!}{n!} \mathrm{Pf}(\Omega)$$

(see [3, vol. V, pp. 417–420]).

AMS (MOS) subject classifications (1970). Primary 57D30, 53C99.

Key words and phrases. Codimension-one foliation, Gaussian curvature, constant curvature.

[1] Research supported by the National Science Foundation.