

describing an ensemble of  $k$  particles. Suppose that we are not allowed to observe trajectories directly, but only to observe the position of  $k$  particles at one fixed time. (Then we know that the predictions of the stochastic interpretation agree with the predictions of quantum mechanics.) We are free to impose any time-dependent potentials we wish and to consider  $k - 1$  of the particles as observing instruments. How much information can we obtain about the trajectory of the remaining particle in this way?

The stochastic interpretation gives a clear meaning to the notion of the probability that a particle (in a process corresponding to a solution of the Schrödinger equation) is ever in a given region during a given interval of time. The orthodox theory of quantum mechanical measurement is restricted to observations made at one fixed time. Is there a quantum mechanical definition of this probability which agrees with the probability given by the stochastic interpretation?

There remains the problem of developing a stochastic relativistic theory. Theories of relativistic interaction appear to require fields. In recent years probabilistic techniques have played a large role in constructive quantum field theory, but the random fields have been constructed on Euclidean space, rather than Minkowski space, and the results for quantum fields have been obtained by analytic continuation. This is analogous to studying the Schrödinger equation by means of the corresponding heat equation, and then analytically continuing in time. The field-theoretic analogue of the stochastic interpretation of the Schrödinger equation remains to be constructed.

ADDED IN PROOF. Some of the questions raised here have been answered by David Shucker in a Princeton thesis (to appear).

#### REFERENCES

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EDWARD NELSON

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*Grundzüge der universellen Algebra*, by Herbert Lugowski, Teubner-Texte zur Mathematik, B. G. Teubner Verlagsgesellschaft, Leipzig, 1976, 238 pp., DM 19.50.

Universal algebra, as a method, has been extremely fruitful; by contrast, as an independent discipline it appears a little arid, owing to the fact that so many of its results have been somewhat less universal in their application. Perhaps the subject has developed best when working in harness with another part of mathematics, such as logic or category theory, and this is reflected in more recent books such as [1], [2]. Another field which would provide a good