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Wittgenstein's lectures on the foundations of mathematics, Cambridge, 1939, from the notes of R. G. Bosanquet, Norman Malcolm, Rush Rhees, and Yorick Smythies, edited by Cora Diamond, Cornell Univ. Press, Ithaca, New York, 1976, 300 pp., \$18.50.

W, the favorite pupil of Bertrand Russell, became famous after World War I for a slim volume *Tractatus logico-philosophicus*. Apart from the obviously impressive flair and vigor of the style the book remains an outstanding example of the heroic tradition of Western philosophy, with its questions about the general structure of knowledge or the correct analysis of (all meaningful) propositions. Since the questions certainly occur, to anyone, prior to any detailed intellectual experience, more or less the same is expected of the answers. *Tractatus* is quite remarkable in this respect: no appeal to anything that would ordinarily be called a discovery, about physical or mental phenomena, barely any use of new intellectual let alone material tools except—of all things—truth tables for propositional logic. Offstage there was a discovery, chemical atomism: the correct analysis of substances in terms of atoms and the chemical bonds between them. The general idea of *Tractatus* is that there is a (tacitly:finite) supply of simples corresponding to atoms, and so-called elementary propositions about them, independent of each other, as in elementary probability theory, but in contrast to chemical bonding. Arbitrary propositions are then to be analyzed in terms of elementary ones. Sense corresponds to chemically possible combinations of atoms. Actually there was very little about mathematics in *Tractatus* except for a brief reference to an operational analysis—in contrast to the set-theoretic analysis in *Principia*, which is also in the heroic tradition, but a less pure example.

In the decade after *Tractatus* was completed, W turned away from academic life, renounced an immense fortune, and did other romantic things. (He also retained, to the end of his life, a freshness of mind quite unusual even among those not desiccated by an academic atmosphere nor preoccupied with finance.) During that period he became disillusioned with *Tractatus*; so much so that a perfectly trivial objection by a friend triggered his decision to spell out his misgivings, not so much about his personal contribution as about the whole heroic tradition. (The objection concerned the inadequacy of *Tractatus* for a correct analysis of some expressive Italian gesture—as if this were a principal weakness of *Tractatus*.) W found that quite elementary mathematics provided excellent illustrations of weaknesses of traditional foundations, t.f. for short.

Trivially, W's revised views are 'revolutionary' for t.f.; but they are quite close to those of many thoughtful mathematicians, for example, in Bourbaki's most interesting (and no longer well-known) manifesto: *L'architecture des mathématiques* [1].¹ A principal problem was to put those views into words convincingly, and W was aware of this fact; thus the last sentence of the book under review stresses [*The seed I am most likely to sow is*] a certain jargon; cf. [2]. The main aim of this review is to restate the complaints of W and

¹ [n] means the *n*th note at the end of this essay, containing historical and bibliographical details.