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BOOK REVIEWS

A comprehensive introduction to differential geometry, by Michael Spivak, Publish or Peril, Inc., Boston, Mass., volume 3, 1975, 474 + ix pp., $16.25; volume 4, 1975, v + 561 pp., $17.50; volume 5, 1975, v + 661 pp., $18.75.

Spivak's *Comprehensive introduction* takes as its theme the classical roots of contemporary differential geometry. Spivak explains his Main Premise (my term) as follows: "in order for an introduction to differential geometry to expose the *geometric* aspect of the subject, an historical approach is necessary; there is no point in introducing the curvature tensor without explaining how it was invented and what it has to do with curvature". His second premise concerns the manner in which the historical material should be presented: "it is absurdly inefficient to eschew the modern language of manifolds, bundles, forms, etc., which was developed precisely in order to rigorize the concepts of classical differential geometry".

Here, Spivak is addressing "a dilemma which confronts anyone intent on penetrating the mysteries of differential geometry". On the one hand, the subject is an old one, dating, as we know it, from the works of Gauss and Riemann, and possessing a rich classical literature. On the other hand, the rigorous and systematic formulations in current use were established relatively recently, after topological techniques had been sufficiently well developed to provide a base for an abstract global theory; the coordinate-free geometric methods of E. Cartan were also a major source. Furthermore, the viewpoint of global structure theory now dominates the subject, whereas differential geometers were traditionally more concerned with the local study of geometric objects.

Thus it is possible and not uncommon for a modern geometric education to leave the subject's classical origins obscure. Such an approach can offer the great advantages of elegance, efficiency, and direct access to the most active areas of modern research. At the same time, it may strike the student as being frustratingly incomplete. As Spivak remarks, "ignorance of the roots of the subject has its price–no one denies that modern formulations are clear, elegant and precise; it's just that it's impossible to comprehend how any one ever thought of them."

While Spivak's impulse to mediate between the past and the present is a natural one and is by no means unique, his undertaking is remarkable for its ambitious scope. Acting on its second premise, the *Comprehensive introduction* opens with an introduction to differentiable manifolds; the remaining four volumes are devoted to a geometric odyssey which starts with Gauss and Riemann, and ends with the Gauss-Bonnet-Chern Theorem and characteristic classes. A formidable assortment of topics is included along the way, in which we may distinguish several major historical themes:

In the first place, the origins of fundamental geometric concepts are investigated carefully. As just one example, Riemannian sectional curvature is introduced by a translation and close exposition of the text of Riemann's remarkable paper, *Über die Hypothesen, welche der Geometrie zu Grunde*