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EQUIVARIANT SMOOTHING THEORY

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Given a finite group G acting on a topological manifold M, when can we put a smooth structure on M such that G acts smoothly? Our approach to this problem is via equivariant immersion theory. This generalizes the immersion theory approach of [12], and we begin by reviewing these ideas. Details will appear in [13].

1. The immersion approach to smoothing theory. A map $\alpha: M_1^n \to M_2^n$ between n-dimensional topological manifolds is called a (topological) *immersion* if α is a local homeomorphism. Of course, a smooth immersion is a topological immersion of the underlying topological manifolds. The basis of the immersion approach to smoothing is the following trivial lemma:

LEMMA 1. *A topological immersion α of a topological manifold M^n into a smooth manifold V^n defines a unique smooth structure on M such that α becomes a smooth immersion.*

In fact, define smooth local coordinates on M by pulling back the local coordinates on V via the local homeomorphisms. We will denote this smooth structure by M_α.

Recall that the differential of a smooth immersion $f: V_1^n \to V_2^n$ induces a bundle homomorphism $df: TV_1 \to TV_2$ of the tangent vector bundles which is an isomorphism on fibres. Call such a bundle homomorphism a representation and let $R(TV_1, TV_2)$ be the space of representations with the C^0-topology and $I^\infty(V_1, V_2)$ the space of smooth immersions with the C^∞-topology. The Smale-Hirsch theorem for manifolds of the same dimension states:

THEOREM A (HIRSCH). *If no component of V_1 is closed, $d: I^\infty(V_1, V_2) \to R(TV_1, TV_2)$ is a weak homotopy equivalence. The relative version for immersions modulo a given immersion on a neighborhood of a closed subset A holds, provided $\overline{M - A}$ has no compact components.*

For a topological manifold M we have Milnor's tangent microbundle [15], [12]. Since the fibre of τM over $p \in M$ is essentially a neighborhood germ, a local homeomorphism $f: M_1 \to M_2$ defines a microbundle representation

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