BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 6, November 1977

ON THE SINGULARITIES OF NONLINEAR FREDHOLM OPERATORS

BY M. S. BERGER[1] AND R. A. PLASTOCK

Communicated by Ronald G. Douglas, June 8, 1977

Let f be a C^1 mapping between two Banach spaces X and Y. Then a key problem of functional analysis consists of describing the precise structure of the solutions of the equation $f(x) = y$, as y varies over Y under realistic hypotheses on the map f.

Motivated by the study of boundary value problems for systems of nonlinear elliptic partial differential equations we suppose the map f is a C^1 proper nonlinear Fredholm operator of nonnegative index acting between two *infinite dimensional* Banach spaces X and Y. Thus for each $x \in X$, $f'(x)$ is a linear Fredholm operator, index $f =$ index $f'(x)$ and $f^{-1}(K)$ is compact for each compact $K \subset Y$ (cf. Smale [7]).

In order to study the change in the structure of the solutions of $f(x) = y$ as y varies over y, we single out the singular points of f (i.e. the points x at which $f'(x)$ is not surjective). Known results study such mappings f with no singularities: for example, Banach and Mazur [1] assert that if f has index zero, f is a global homeomorphism; and a generalization to higher index by Earle and Eells [3].

Novel features of our results are their specifically infinite dimensional nature, the remarkable distinction they demonstrate between maps zero and positive Fredholm index, and their applicability to yield sharp results on the range of nonlinear elliptic systems (when combined with local bifurcation theory).

The main abstract results. Thus let f be a proper Fredholm nonlinear operator of index $p \geqslant 0$ as above. Let B denote the set of singular points of f and $S = f(B)$ denote the singular values of f. Then the following results hold:

THEOREM 1 (STRUCTURE THEOREM). *The solutions of $f(x) = y$ are either empty or homeomorphic compact p dimensional manifolds on each component of $Y - S$. In particular, for $p = 0$ the number of solutions of $f(x) = y$ are the same finite number on each component of $Y - S$.*

THEOREM 2 (REMOVABLE SINGULARITY THEOREM). *Suppose X and Y*

AMS (MOS) subject classifications (1970). Primary 47H15, 35J60.

[1] Research partially supported by an NSF grant.