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AFFINE PI RINGS ARE CATENARY

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An affine PI ring, is a ring satisfying a polynomial identity, such that the ring is finitely generated as an algebra over a central subfield k. A long outstanding question in the theory of these rings, was whether any two saturated (increasing) chains of primes beginning at one common prime, must have the same length. (See for e.g. Procesi [3, p. 186].) We answer this question in the affirmative.

THEOREM 1. *If R is a prime affine PI ring then* $\dim R = \dim R/P + \mathrm{ht}\, P$, *for every prime P.*

Here $\dim R$ means the length of the longest chain of nonzero primes (i.e. Krull dimension), and $\mathrm{ht}\, P$ is the length of the longest chain of nonzero primes contained in P.

Basic to the proof is passage to the ring $R[T]$, i.e. the ring R with the coefficients of the characteristic polynomials of elements of R adjoined. (R has a central simple quotient ring RK, so think of $\mathrm{tr}(r) \in RK$.) In [4] we proved that $R[T]$ was an integral extension of R, if R is noetherian (in fact a finite R module) and were able to lift our primes to $R[T]$ and prove the result there. If R is not noetherian, we do not have a surjection from Spec $R[T]$ to Spec R, but we are able to lift ht 1 primes.

LEMMA. *If R is prime affine PI and $P \subsetneq R$ is a prime with* $\mathrm{ht}\, P = 1$, *then there exists a P' lying over it in $R[T]$.*

The reason we can do this is $R[T]$ is contained in a finite R module: $k[T]$ is affine and is contained in the complete integral closure of Centre (R) [4, Corollary 2, Theorem 2].

Actually we have a stronger result than in [4], namely:

THEOREM 2. *If R is prime PI and c is the evaluation of a central polynomial, then $c^m R[T] \subset R$ for some m.*

The proof of this is based on the fact that $R[c^{-1}]$ is Azumaya and so $R[T] \subseteq R[c^{-1}]$.

Next we find an irreducible variety W containing $V(P')$, such that W is not

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