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FINITE GROUPS VIEWED LOCALLY

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1. Introduction. Groups have now been studied for over two hundred years. Their first great success, over one hundred and forty years ago, was Galois theory which exploited the relationship between polynomial equations and finite groups. This idea motivated further developments. For example, Lie groups were introduced to study differential equations in an analogous way.

A common theme in many parts of group theory has been the classification of simple groups. This was achieved for complex Lie groups by Killing and for real Lie groups by Cartan. Similar results for algebraic groups were obtained in work led by Chevalley. Only now does it seem likely that a classification can be accomplished for finite simple groups. This is a more complicated and difficult problem by far than any of the other classifications. The important applications of our present knowledge of finite simple groups will be followed by many more such results.

Apart from the basic ideas about normal structure, in particular, normal subgroups, homomorphisms, quotient groups, and direct products, there are three separate but intertwined methods of studying the structure of finite groups. First, we have the oldest: permutation groups. One studies the G-sets for a group G, the sets that G acts on. This leads to all sorts of geometrical and combinatorial considerations. Second, there is the method of representation theory where one deals with G-modules, namely vector spaces, abelian groups and modules on which the group G acts. Here one gets involved with characters and then algebras, number fields, division algebras, non-semi-simple rings and homological algebra. Third, there is the local method, that part of the subject that begins with the Sylow theorems.

And this is our subject. We shall have a glimpse and a survey of the local method emphasizing the results that are so useful in studying simple groups. We shall not explore the other methods and we shall only touch on the vast web of relationships between these different methods. However, it is the way that these disparate methods so perfectly complement each other which is the basis for the tremendous progress on classifying finite simple groups.

We now come to the basic definitions. If G is a finite group and p is a prime then a subgroup of G of order a power of p is called a p-subgroup of G. If Q is a nonidentity p-subgroup of G then the normalizer $N(Q)$ is called a p-local subgroup of G, or simply a local subgroup of G. It is the p-local

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