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STOKES PHENOMENA

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1. Introduction. Let x be a complex variable, and set $\mathcal{D}(R) = \{x; |x| > R\}$. Consider two differential equations

$$(\mathcal{E}_1) \qquad dy/dx = A_1(x)y$$

and

$$(\mathcal{E}_2) \qquad du/dx = A_2(x)u,$$

where y and u are n-dimensional vectors, and A_1 and A_2 are n-by-n matrices whose components are holomorphic in $\mathcal{D}(R_1)$ and $\mathcal{D}(R_2)$ respectively and meromorphic at $x = \infty$. Note that if A_1 and A_2 are replaced by their Laurent expansions at $x = \infty$, then $(\mathcal{E}_1)$ and $(\mathcal{E}_2)$ become two differential equations defined in a vector space over the field of quotients of the ring of formal power series in x^{-1}. The two equations $(\mathcal{E}_1)$ and $(\mathcal{E}_2)$ are said to be *formally equivalent* if there exists an n-by-n matrix $T(x)$ such that (i) $T(x) = x^q\Sigma_{h=0}^{\infty} T_h x^{-h}$ is a formal series (with an integer q and constant matrices T_h), (ii) $\det T(x) \neq 0$ as a formal series, and (iii) the transformation $u = T(x)y$ reduces $(\mathcal{E}_1)$ to $(\mathcal{E}_2)$ formally. The two equations $(\mathcal{E}_1)$ and $(\mathcal{E}_2)$ are said to be *meromorphically equivalent* if there exists an n-by-n matrix $T(x)$ such that (i) the components of $T(x)$ are holomorphic in $\mathcal{D}(R_3)$ for some R_3 and meromorphic at $x = \infty$, (ii) $\det T(x) \neq 0$ in $\mathcal{D}(R_3)$, and (iii) the transformation $u = T(x)y$ reduces $(\mathcal{E}_1)$ to $(\mathcal{E}_2)$ in $\mathcal{D}(R_3)$.

Let $\mathfrak{A}$ be an equivalence class of such differential equations with respect to the formal equivalence. The main concern in this report is to characterize the equivalence classes with respect to the meromorphic equivalence within $\mathfrak{A}$.

2. Asymptotic properties of an analytic fibre bundle. For a sector S, set $S(R) = \{x \in S; |x| > R\}$. An n-by-n matrix $F(x)$ is said to have an asymptotic expansion $A_{sp}(F)$ as x tends to ∞ in S, if (i) the components of $F(x)$ are holomorphic in $S(R)$ for some R, (ii) $A_{sp}(F) = x^q\Sigma_{h=0}^{\infty} G_h x^{-h}$ is a formal series, and (iii) for every positive integer N there exists a positive number K_N such that $|F(x) - x^q\Sigma_{h=0}^{N} G_h x^{-h}| \leq K_N|x|^{q-N-1}$ in $S(R)$. For an n-by-n matrix $F(x)$, the notation $F(x) \in A(S(R))$ means that (i) $F(x)$ is holomorphic

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