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BIFURCATION OF PERIODIC ORBITS ON MANIFOLDS, AND HAMILTONIAN SYSTEMS[1]

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We consider a vector field X_0 having a whole submanifold $\Sigma \subset M$ of periodic points, and ask if any periodic orbits persist under small perturbation, i.e. do all vector fields Y sufficiently near X_0 have periodic orbits lying near Σ. Σ is assumed to be compact. Although in the general case there are simple counterexamples (e.g. on $\Sigma = n$ torus) some natural hypotheses on Σ and the flow of X_0 guarantee periodic orbits for Y, which are thought of as bifurcating off the manifold Σ. Our method here is closely analogous to that of Moser [2], [3], and also his method of averaging on manifolds [1].

In the case of Hamiltonian flows, these methods take on added significance, and the classical action integral makes an appearance. Here the results may be viewed as an extension to S^1-actions of results of Weinstein carried out for Z_n-actions [4], [5].

1. The general case. Let X_0 be a vector field on a manifold M and ϕ^t its induced flow. A nondegenerate periodic manifold of X_0 of period τ is a ϕ^t-invariant submanifold of M such that $\phi^\tau(z) = z$ for all $z \in \Sigma$, and such that 1 is an eigenvalue of $d\phi_z^\tau$ of algebraic multiplicity $k = \dim \Sigma$.

We denote the space of vector fields over M by $X(M)$, having the usual C^k norm $\|\cdot\|_k$. We parametrize a neighborhood of the identity in Diff(M) by a neighborhood of $0 \in X(M)$ by taking a metric and setting $u(z) = \exp_z U(z)$, for $U \in X(M)$ small enough. We define an operator $P(u)$: $X(M) \longrightarrow X(M)$ which transports vectors at z to vectors at $u(z)$ by setting, for $W \in T_zM$,

$$P(u)W = \frac{d}{dh}\bigg|_{h=0} \exp_z(U(z) + hW).$$

LEMMA A. *Let X_0 be a C^{l+1} vector field on M^n generating the flow ϕ^t, having a compact nondegenerate periodic manifold Σ of period* 1. *Suppose Y is a vector field so that $\|Y - X_0\|_{l+1} < \epsilon$ in some neighborhood of Σ. Then for ϵ sufficiently small, there exists a C^l vector field $V \in X(\Sigma)$, a C^l embedding u: $\Sigma \longrightarrow M$ near the inclusion, and ϕ^t-invariant function λ: $\Sigma \longrightarrow R$ so that*

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