BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 5, September 1977

THE NUMBER OF SOLUTIONS TO THE CLASSICAL PLATEAU PROBLEM IS GENERICALLY FINITE

BY R. BÖHME AND A. J. TROMBA

Communicated by S. S. Chern, January 21, 1977

0. Introduction. The question of how many solutions there are to the classical problem of Plateau has been open for roughly a century. Existence of at least one solution was proved in 1931 independently by T. Rado [7] and J. Douglas [3]. Courant, in his book *Dirichlet's principle, conformal mappings and minimal surfaces* [2], outlines an argument which suggests that there may exist rectifiable curves in $\mathbf{R}^3$ bounding on uncountable number of solutions. It has been believed for some time that for all sufficiently nice curves there are only a finite number of surfaces of mean curvature zero which they bound. In this note we state that there exists an open dense set of curves which bound a finite number of classical minimal surfaces of the type of the two disc. This result essentially is a synthesis of the ideas of [1], [9], [10].

I. Formulation of results. Let $\alpha\colon S^1 \longrightarrow \mathbf{R}^n$ be a C^∞ embedding of S^1 into $\mathbf{R}^n$, where S^1 denotes the boundary of the open disc $\mathcal{D}$ in $\mathbf{R}^2$. Let $\Gamma^\alpha = \alpha(S^1)$ denote its image.

DEFINITION. A classical solution to Plateau's problem for α is a map u from $\overline{\mathcal{D}}$ into $\mathbf{R}^n$ satisfying the following properties.

(i) $u \in C^0(\overline{\mathcal{D}}) \cap C^\infty(\mathcal{D})$,

(ii) $\Delta u = 0$,

(iii) $\partial u/\partial x \cdot \partial u/\partial y = 0 \ \forall (x, y) \in \mathcal{D}$,

(iv) $\|\partial u/\partial x\| = \|\partial u/\partial y\| \ \forall (x, y) \in \mathcal{D}$,

(v) $u\colon S^1 \longrightarrow \Gamma^\alpha$ homeomorphically.

REMARK.1. By well-known regularity results for minimal surfaces first proved by Hildebrandt [5], and then later improved by Nitsche [6], Heinz and Tomi [4], and Tomi [8], $\alpha \in C^\infty$ implies $u \in C^\infty(\overline{\mathcal{D}})$.

Let $\mathcal{A}$ denote the space of all C^∞ embeddings of S^1 into $\mathbf{R}^n$ with the C^∞ topology.

THEOREM 1. *There exists an open and dense subset $\mathcal{A}_0 \in \mathcal{A}$ such that for all $\alpha \in \mathcal{A}_0$ there are only finitely many classical solutions of the Plateau problem.*

REMARK 2. These finitely many surfaces are nondegenerate in the sense described in [1].

AMS (MOS) subject classifications (1970). Primary 53A10, 65N99, 58E99.