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THE EXISTENCE OF MINIMAL IMMERSIONS OF TWO-SPHERES

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In this article we announce a series of results on the existence of harmonic maps from surfaces to Riemannian manifolds and, as corollaries of these results, obtain theorems on the existence of minimal immersions of 2-spheres.

Let N be a compact connected Riemannian manifold and, for convenience, assume that N is isometrically imbedded in $\mathbf{R}^k$ for some sufficiently large k. Let M be a closed Riemann surface with any metric compatible with its conformal structure. A map $s \in L_1^2(M, \mathbf{R}^k) \cap C^0(M, N)$ is called harmonic if it is an extremal map of the energy integral

$$E(s) = \int_M |ds|^2 \, d\mu_M = \int_M \operatorname{trace} I(x) \, d\mu_M$$

where

$$I(x) = \sum_{i=1}^{k} ds^i \otimes ds^i(x) \in T_x^*(M) \otimes T_x^*(M).$$

Harmonic maps satisfy an Euler-Lagrange equation

$$\Delta s + A(s)(ds, ds) = 0$$

in a weak sense, where A is the second fundamental form of the imbedding $N \subset \mathbf{R}^k$. It then follows from regularity theorems that harmonic maps are C^∞. If s is harmonic and a conformal immersion, it is also an extremal for the area integral.

Proving the existence of harmonic maps of M into N by direct methods from global analysis such as Morse theory or Ljusternik-Schnirelman theory applied to E defined on some function space manifold is difficult, because E is invariant under the conformal group of M, and the extremal maps of E form a noncompact set when $M = S^2$. In particular, E does not satisfy condition C of Palais-Smale. However, for $\alpha > 1$, a slightly different integral,

$$E_\alpha(s) = \int_M (1 + |ds|^2)^\alpha \, d\mu_M$$

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