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CENTRAL SIMPLE ALGEBRAS WITH INVOLUTION

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We will carry the following hypotheses throughout this paper: F is a field of characteristic $\neq 2$; A is a *central simple* F-algebra, i.e. a simple F-algebra of finite dimension, with center F; A has an *involution* $(*)$ of *first kind*, i.e. an anti-automorphism of degree 2 which fixes the elements of F. The classic reference on central simple algebras is [1], which also treats involutions.

The dimension of A (over F) must be a perfect square, which we denote as n^2. A famous conjecture is that A must be a tensor product of a matrix subalgebra (over F) and quaternion subalgebras (over F); since the conjecture is easily proved when $n < 8$, the first case of interest is when $n = 8$. The main theorem of this paper is the following result when A is a division algebra.

MAIN THEOREM. *If $n = 8$, then A has a maximal subfield which is a Galois extension over F, with Galois group $\mathbf{Z}_2 \oplus \mathbf{Z}_2 \oplus \mathbf{Z}_2$.*

The proof relies heavily on a computational result of Rowen and Schild, which will be given below. Before sketching the proof of the main theorem, we start with some general results (true for any n), which can be verified easily.

PROPOSITION 1. *Given a subfield K of A containing F, we have an involution of A (of the first kind), which fixes the elements of K.*

PROPOSITION 2. *Suppose A is also a division algebra. Suppose K is a nonmaximal subfield of A (containing F), with an automorphism φ over F, having degree 2. Then $\varphi^*\colon K \longrightarrow K^*$ can be given by conjugation in A, by an element which is symmetric (resp. antisymmetric) with respect to $(*)$.*

Let $\overline{F}$ denote the algebraic closure of F, and let $M_n(\overline{F})$ be the algebra of matrices over $\overline{F}$. Then $(*)$ induces an involution on $M_n(\overline{F}) \approx A \otimes_F \overline{F}$, given by $(\Sigma a_i \otimes \beta_i)^* = \Sigma a_i^* \otimes \beta_i$, for $a_i \in A$ and $\beta_i \in F$. We say $(*)$ is of *symplectic type* if the extension of $(*)$ to $M_n(\overline{F})$ is symplectic, i.e. *not* cogredient to the transpose (of matrices), cf. [1, p. 155]. Such an involution exists iff n is even, in which case we can build a "universal" F-algebra with symplectic type involu-

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