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COVERING SMOOTH HOMOTOPIES OF ORBIT SPACES

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1. **Introduction.** Let G be a compact Lie group. If M is a G-space, then one may consider M as a sort of singular fiber bundle over the orbit space M/G. Palais has proved a covering homotopy theorem and constructed classifying spaces for such types of "bundles" [8]. In this note we describe an analogue of Palais' covering homotopy theorem which is valid for smooth G-actions. Various guises of this smooth Palais theorem have already been important in the study of regular actions of the classical groups (see [3], [4], [5] and references therein).

2. **Orbit spaces.** Let X and Y be smooth ($= C^\infty$) G-manifolds. A real-valued function on X/G is said to be smooth if it lifts to a smooth function on X, so $C^\infty(X/G) \simeq C^\infty(X)^G$. F: $X/G \longrightarrow Y/G$ is said to be smooth if $F^*C^\infty(Y/G) \subseteq C^\infty(X/G)$. By the slice representation at $x \in X$ we mean the representation of the isotropy group G_x on the normal bundle at x to the orbit Gx. Two G-orbits are said to have the same normal type if they contain points with the same isotropy group and isomorphic slice representations (up to trivial factors). The subsets of orbits of given normal type give a stratification of X/G by C^∞ manifolds. One can show that X/G is locally homeomorphic to closed semialgebraic sets where the homeomorphisms can be chosen to induce isomorphisms on rings of C^∞ functions and, up to components, to preserve strata ([1], [9]). Here a closed semialgebraic set is given its canonical stratification by singularity and the ring of C^∞ functions induced from the ambient space.

If $\xi \in X/G$, let $T_\xi(X/G)$ denote the Zariski tangent space at ξ, i.e. the dual of $M_\xi/(M_\xi)^2$ where M_ξ is the maximal ideal of the ring of germs of smooth functions at ξ. $T_\xi(X/G)$ is always finite dimensional. Let σ_ξ denote the stratum of X/G containing ξ, and let $N_\xi(X/G)$ denote the normal space $T_\xi(X/G)/T_\xi(\sigma_\xi)$. Der $C^\infty(X/G)$ denotes the real-linear derivations of $C^\infty(X/G)$, and we call elements of Der $C^\infty(X/G)$ smooth vector fields on X/G. $B \in$ Der $C^\infty(X/G)$ is said to be strata preserving if for each $\xi \in X/G$ the associated tangent vector $B(\xi)$ lies in $T_\xi(\sigma_\xi)$. $\mathfrak{X}^\infty(X/G)$ denotes the strata preserving elements of Der $C^\infty(X/G)$, and $\mathfrak{X}^\infty(X)$ denotes the smooth vector fields on X.

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