BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 5, September 1977

Rational methods in Lie algebras, by George B. Seligman, Lecture Notes in Pure and Applied Math., vol. 17, Marcel Dekker, New York and Basel, 1976, viii + 346 pp., \$29.50.

The simple Lie algebras over **C** were classified by W. Killing and E. Cartan just prior to 1900. These comprise four infinite families A_n, B_n, C_n, D_n, along with five exceptional algebras E_6, E_7, E_8, F_4, G_2, the subscript denoting the rank. The infinite families occur as Lie algebras of the "classical" complex Lie groups, e.g., A_n is the Lie algebra of the special linear group $\mathrm{SL}(n + 1, \mathbf{C})$. The exceptional Lie algebras are intimately related to certain other exceptional structures arising in nonassociative algebra, such as 8-dimensional Cayley algebras (octonions) and nonspecial 27-dimensional simple Jordan algebras.

In a nutshell, the classification amounts to characterizing each simple Lie algebra L by a finite configuration of vectors in a euclidean space $\mathbf{R}^n$ (the *root system*, consisting of certain linear functions on a Cartan subalgebra of L), subject to some constraints on the angles between vectors; the determination of all possible root systems is then a sophisticated exercise in euclidean geometry. Of course, one also has to prove the existence of algebras of all types A–G.

Actually, it is the *real* Lie groups and their Lie algebras which lie closer to the sources of Lie theory in differential geometry. If L is a simple Lie algebra over **R**, there are just two possibilities:

(1) The complexification $L_{\mathbf{C}} = \mathbf{C} \otimes_{\mathbf{R}} L$ may be a direct sum of two simple Lie algebras, interchanged by complex conjugation, in which case L is isomorphic to either of these viewed as Lie algebra over **R**; the classification of these real simple algebras leads back directly to the list A–G.

(2) $L_{\mathbf{C}}$ may be simple, in which case L is called a *real form* of $L_{\mathbf{C}}$. Here the type A–G of $L_{\mathbf{C}}$ is called the *split type* of L; each split type turns out to embrace two or more distinct possibilities for L.

By 1914 Cartan [4] had obtained the complete classification of simple Lie algebras over **R**, from a "compact" point of view, exploiting the fact that each simple Lie algebra over **C** has a unique compact real form (corresponding to a compact Lie group). Later there were other treatments in this vein by F. Gantmacher [5], S. Murakami [10], as well as treatments from an "anticompact" point of view, cf. S. Araki [2], I. Satake [11] (appendix by M. Sugiura). In either case, the essential idea is to compare the unknown L with the known $L_{\mathbf{C}}$. This amounts to a determination of the nonabelian galois cohomology set $H^1(\mathrm{Gal}(\mathbf{C}/\mathbf{R}), \mathrm{Aut}\, L_{\mathbf{C}})$.

It was not long before attempts were made to classify simple Lie algebras over other fields of characteristic 0, motivated by the fact that the classical linear groups (special linear, orthogonal, symplectic) exist over general fields. In the 1930's, W. Landherr [8], [9], a student of E. Artin, made some progress in this direction. N. Jacobson, H. Freudenthal, and others pushed the purely algebraic study of Lie algebras substantially farther and enlarged the study of the exceptional types [7]. (The paper [6] furnished the title of the book under review, though it was concerned only with structural questions, not classification.)