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COEFFICIENTS OF UNIVALENT FUNCTIONS

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The interplay of geometry and analysis is perhaps the most fascinating aspect of complex function theory. The theory of univalent functions is concerned primarily with such relations between analytic structure and geometric behavior.

A function is said to be *univalent* (or *schlicht*) if it never takes the same value twice: $f(z_1) \neq f(z_2)$ if $z_1 \neq z_2$. The present survey will focus upon the class S of functions

$$f(z) = z + a_2 z^2 + a_3 z^3 + \cdots$$

analytic and univalent in the unit disk $|z| < 1$. This is the class of all univalent functions normalized by the conditions $f(0) = 0$ and $f'(0) = 1$. We shall concentrate on coefficient problems for the class S and for related classes, with emphasis on recent results and open problems. Most of the methods we shall describe have wide scope and are not restricted to coefficient problems.

The theory of univalent functions is an old but very active field. The last ten or fifteen years have witnessed a number of major advances. In fact, progress has been so rapid that Hayman's 1965 survey **[66]** of coefficient problems is already rather out of date.

In most general form, the *coefficient problem* is to determine the region of $\mathbb{C}^{n-1}$ occupied by the points $(a_2, \ldots, a_n)$ for all $f \in S$. The deduction of such precise analytic information from the geometric hypothesis of univalence is exceedingly difficult. We shall confine attention to the more special problem of estimating $|a_n|$, the modulus of the nth coefficient. Even this problem has never been solved completely.

1. The Bieberbach conjecture. The leading example (aside from the identity) of a function of class S is the *Koebe function*

$$k(z) = z(1-z)^{-2} = z + 2z^2 + 3z^3 + \cdots,$$

which maps the unit disk onto the full plane slit along the negative real axis from $-1/4$ to ∞. The Koebe function and its rotations $k_\varphi(z) = e^{-i\varphi}k(e^{i\varphi}z)$ have long been known (see **[55]**, **[63]**, **[120]**, **[38]**) to maximize and minimize

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