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A TRUNCATION PROCESS FOR REDUCTIVE GROUPS

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Let G be a reductive group defined over $\mathbf{Q}$. Index the parabolic subgroups defined over $\mathbf{Q}$, which are standard with respect to a minimal ${}^{(0)}P$, by a partially ordered set $\mathfrak{J}$. Let 0 and 1 denote the least and greatest elements of $\mathfrak{J}$ respectively, so that ${}^{(1)}P$ is G itself. Given $u \in \mathfrak{J}$, we let ${}^{(u)}N$ be the unipotent radical of ${}^{(u)}P$, ${}^{(u)}M$ a fixed Levi component, and ${}^{(u)}A$ the split component of the center of ${}^{(u)}M$. Following [1, p. 328], we define a map ${}^{(u)}H$ from ${}^{(u)}M(\mathbf{A})$ to ${}^{(u)}\mathfrak{a} = \operatorname{Hom}(X({}^{(u)}M)_{\mathbf{Q}}, \mathbf{R})$ by

$$e^{\langle \chi, {}^{(u)}H(m)\rangle} = |\chi(m)|, \qquad \chi \in X({}^{(u)}M)_{\mathbf{Q}},\ m \in {}^{(u)}M(\mathbf{A}).$$

If K is a maximal compact subgroup of $G(\mathbf{A})$, defined as in [1, p. 328], we extend the definition of ${}^{(u)}H$ to $G(\mathbf{A})$ by setting

$${}^{(u)}H(nmk) = {}^{(u)}H(m), \qquad n \in {}^{(u)}N(\mathbf{A}),\ m \in {}^{(u)}M(\mathbf{A}),\ k \in K.$$

Identify ${}^{(0)}\mathfrak{a}$ with its dual space via a fixed positive definite form $\langle\ ,\ \rangle$ on ${}^{(0)}\mathfrak{a}$ which is invariant under the restricted Weyl group Ω. This embeds any ${}^{(u)}\mathfrak{a}$ into ${}^{(0)}\mathfrak{a}$ and allows us to regard ${}^{(u)}\Phi$, the simple roots of $({}^{(u)}P, {}^{(u)}A)$, as vectors in ${}^{(0)}\mathfrak{a}$. If $v \leqslant u$, ${}^{(v)}P \cap {}^{(u)}M$ is a parabolic subgroup of ${}^{(u)}M$, which we denote by ${}^{(v)}_{(u)}P$ and we use this notation for all the various objects associated with ${}^{(v)}_{(u)}P$. For example, ${}^{(v)}_{(u)}\mathfrak{a}$ is the orthogonal complement of ${}^{(u)}\mathfrak{a}$ in ${}^{(v)}\mathfrak{a}$ and ${}^{(v)}_{(u)}\Phi$ is the set of elements $\alpha \in {}^{(v)}\Phi$ which vanish on ${}^{(u)}\mathfrak{a}$.

Let R be the regular representation of $G(\mathbf{A})$ on $L^2(ZG(\mathbf{Q})\backslash G(\mathbf{A}))$, where we write Z for ${}^{(1)}A(\mathbf{R})^0$, the identity component of ${}^{(1)}A(\mathbf{R})$. Let f be a fixed K-conjugation invariant function in $C_c^\infty(Z\backslash G(\mathbf{A}))$. Then $R(f)$ is an integral operator whose kernel is

$$K(x, y) = \sum_{\gamma \in G(\mathbf{Q})} f(x^{-1}\gamma y).$$

If $u < 1$ and $\lambda \in {}^{(u)}\mathfrak{a} \otimes \mathbf{C}$, let $\rho(\lambda)$ be the representation of $G(\mathbf{A})$ obtained by inducing the representation

$$(n, a, m) \longrightarrow {}_{(u)}R_{\mathrm{disc}}(m) \cdot e^{\langle \lambda, {}^{(u)}H(m)\rangle}$$

from ${}^{(u)}P(\mathbf{A})$ to $G(\mathbf{A})$. Here ${}_{(u)}R_{\mathrm{disc}}$ is the subrepresentation of the representation

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