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SEMIAMARTS AND FINITE VALUES

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Let X_n be a sequence of real-valued random variables adapted to an increasing sequence of σ-algebras F_n. We denote by T, T_f, $\bar{T}$ respectively the collection of bounded, finite, and arbitrary stopping times for $(F_n)_{n\in \mathbf{N}}$. This paper reports on recent progress concerning the theory of *semiamarts,* i.e. processes for which $(EX_\tau)_{\tau\in T}$ is bounded, initiated in [3], and the theory of *amarts*, i.e. processes for which $\lim_{\tau\in T} EX_\tau$ exists. We relate the notion of semiamart to processes of interest in the theory of optimal stopping (cf. [2]), namely X_n such that $|EX_\mu| < \infty$ for $\mu \in T_f$, or for $\mu \in \bar{T}$. For independent random variables X_n and for processes of the form $X_n = c_n^{-1} \Sigma_{i=1}^n Y_i$ with increasing c_n's and independent nonnegative Y_i's, a new dominated estimate

$$E(\sup X_n^+) \leqslant K \sup_{\mu\in\bar{T}} EX_\mu \quad (=KV(\bar{T}))$$

with $K = 2$ in the first and $K < 5.46$ in the second case, shows that such processes are semiamarts if and only if $\sup|X_n|$ is integrable. Also in the case when $F_n = F_m$ for all $n, m \in \mathbf{N}$, a semiamart has a necessarily integrable supremum. This observation is used to construct averages of aperiodic stationary sequences, which are not semiamarts–thereby strengthening a result announced by A. Bellow [1]. This can be done also in the "descending" case, i.e. when the time domain $\mathbf{N}$ is replaced by $-\mathbf{N}$ (see [3]); thus our results indicate that there are no connections between the amart theory and the ergodic theory of point transformations.

THEOREM 1 (RIESZ DECOMPOSITION FOR SEMIAMARTS). *Every semiamart* (X_n, F_n) *can be represented as* $X_n = Y_n + Z_n$ *where* (Y_n, F_n) *is a martingale and* (Z_n, F_n) *is an* L_1*-bounded semiamart such that for each* $A \in \bigcup F_m$

$$\liminf_n \frac{1}{n}\sum_{i=1}^n \int_A Z_i \leqslant 0 \leqslant \limsup_n \frac{1}{n}\sum_{i=1}^n \int_A Z_i.$$

This generalizes the Riesz decomposition for amarts [3]. A variant of Theorem 1 permits us to give necessary and sufficient conditions for the uniqueness of the Riesz decomposition. One consequence of the Riesz decomposition is:

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