BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 4, July 1977

SOME ESTIMATIONS IN THE TOPOLOGY OF SIMPLY-CONNECTED ALGEBRAIC SURFACES

BY B. MOISHEZON

Communicated by R. T. Seeley, November 10, 1976

The simplest nontrivial oriented topological surface is the 2-dimensional torus. It is well known that any compact Riemann surface is topologically equivalent to the 2-sphere with handles attached, that is, to a connected sum of 2-toruses. We can consider this decomposition as corresponding to the canonical decomposition of the (skew-symmetric) intersection form of 1-homologies on the given Riemann surface.

In the case of simply-connected algebraic surfaces the intersection form of 2-homologies plays a fundamental role because it defines completely the homotopy type of the corresponding 4-dimensional topological manifold (see [1], [2]).

Performing a σ-process on the given simply-connected algebraic surface V we obtain an algebraic surface V' which contains a 2-dimensional homology class with self-intersection equal -1 (which is an odd number). Then it is well known (see [3], [4]) that there exists a basis of $H_2(V', \mathbf{Z})$ such that the corresponding intersection matrix is diagonal. The corresponding "elementary blocks" $\|+1\|$ and $\|-1\|$ are the intersection matrices of the simplest nontrivial oriented simply-connected 4-manifolds:

P = complex projective plane with its usual orientation and Q = complex projective plane with orientation opposite to the usual. From the homotopy classification theorem [1], [2], it follows that V' is homotopy equivalent to a connected sum of P's and Q's. Of course, the "ideal situation" (analogous to the mentioned above topological decomposition of compact Riemann surfaces), which we could expect, is the existence of a homeomorphism of V' to this connected sum. However, there are some nondirect indications that V' is homeomorphic to a connected sum of P's and Q's if and only if V' is a rational algebraic surface. (This conjecture was formulated in [5].) The question is still open, but assuming the conjecture we can consider as a realistic aim only the problem of estimating how "far" topologically is the given nonrational simply-connected algebraic surface from an "ideal" topological model, that is, from a connected sum of P's and Q's.

In [6] Wall proved the following theorem: *If M_1, M_2 are simply-connected compact 4-manifolds, which are homotopically equivalent, then there exists*

AMS (MOS) subject classifications (1970). Primary 57A15, 14J99, 32J15.