BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 4, July 1977

MARKOV CELL STRUCTURES

BY F. T. FARRELL AND L. E. JONES[1]

Communicated by S. S. Chern, January 15, 1977

ABSTRACT. We show that the partition underlying a Markov partition for a dynamical system can be chosen to be a cell complex structure.

Let M denote a Riemannian manifold of dimension m, Λ a compact subset of M lying in the interior of M, and $h: M \longrightarrow M$ a diffeomorphism. Recall that Λ is called a *hyperbolic* set for h (see [6]) if

(a) $h: \Lambda \longrightarrow \Lambda$ is a homeomorphism;

(b) $T(M)|_\Lambda$ splits as a direct sum $\xi^u \oplus \xi^s$ of continuous subbundles;

(c) $Dh(\xi^u) = \xi^u$, $Dh(\xi^s) = \xi^s$, Dh is expansive on ξ^u and contractive on ξ^s.

If $\Lambda = M$, then $h: M \longrightarrow M$ is called an Anosov diffeomorphism. It is well known that the bundles ξ^u, ξ^s integrate to give transversal foliations W^u, W^s of M. (See [1].) Locally W^u, W^s decompose M into a cartesian product $\mathbf{R}^k \times \mathbf{R}^l$ where k, l are the dimensions of the leaves in W^u, W^s, and $k + l = m$.

A *cell structure* for W^u, W^s consists of a cell structure C for M, such that each cell $\Delta \in C$ splits as a cartesian product of cells $\Delta = \Delta_u \times \Delta_s$ consistent with the local product structure given M by (W^u, W^s). We further require that if $\Delta \in C$ then each of $\partial\Delta_u \times \partial\Delta_s$, $\Delta_u \times \partial\Delta_s$, $\partial\Delta_u \times \Delta_s$ is a cellular subcomplex of C. Let C^i, j denote the subset of M equal the union of open cells

$$\{\Delta \in C | \dim(\Delta_u) \leqslant i, \dim(\Delta_s) \geqslant j\}.$$

A *Markov cell structure* for an Anosov diffeomorphism $h: M \longrightarrow M$ consists of a cell structure C for (W_u, W_s) satisfying $h^n(C^i, j) \subset C^i, j$ for all i, j and some positive integer n.

THEOREM. *There exist Markov cell structures for every Anosov diffeomorphism.*

REMARKS. (1) A Markov cell structure for $h: M \longrightarrow M$ is also a Markov partition for h, but not vice-versa. The partition sets of M underlying a Markov partition of h, as defined in [5], will generally have nonfinitely generated homology groups.

(2) The theorem generalizes to give a Markov cell structure *near* any hyper-

---

AMS (MOS) subject classifications (1970). Primary 58F15.

[1]Both authors research were partially supported by grants from the National Science Foundation.