BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 4, July 1977

RESEARCH ANNOUNCEMENTS

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CLOSURE THEOREMS FOR SPACES OF ENTIRE FUNCTIONS

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Communicated by Richard R. Goldberg, December 8, 1976

We announce a number of single variable approximation theorems. Our approach is to extend de Branges' basic theory of Hilbert spaces of entire functions [2] to a Banach space setting. The resulting structure is sufficiently rich to provide both new approximation results and a unifying structure for many earlier results on approximation by entire functions which are related to the Bernstein approximation problem, for example, Akutowitz [1], Koosis [3], Levinson and McKean [5], Mergelyan [6], Pitt [7] and Pollard [8].

Let C_c be the space of continuous complex functions $m(\lambda)$ on R^1 with compact support and the supremum norm $|m|$. B denotes a fixed Banach function space on R^1 with (semi) norm $\|f\|$. We assume that

(1) $C_c \cap B$ is dense in B, and

(2) The multiplication operator $(m, f) \longrightarrow m(\lambda)f(\lambda)$ is jointly continuous from $C_c \times B$ into B.

Examples of spaces satisfying (1) and (2) are L^p spaces, Orlicz spaces, Lorentz spaces $L_{(p,q)}$ and spaces of continuous functions with weighted supremum norms. Because of condition (2) it follows that for $f \in B$ and $e \in B^*$, the linear functional on C_c given by $m \longrightarrow \langle mf, e\rangle$ is expressible in the form $\langle mf, e\rangle = \int m(\lambda)d\mu_{f,e}$ where $\mu_{f,e}$ is a unique finite Radon measure. The discrete spectrum $\sigma_d(B)$ of B is the set $\{\lambda : |\mu_{f,e}\{\lambda\}| > 0 \text{ for some } f \in B \text{ and } e \in B^*\}$.

Contained in B we fix a linear space H of entire functions with closure $\overline{H}$. We assume for $\operatorname{Im} z \neq 0$ and for f and g in H that the function

$$F(\lambda) \equiv (z - \lambda)^{-1}\{f(z)g(\lambda) - g(z)f(\lambda)\} \in H. \tag{3}$$

If H is closed under the conjugation $h \longrightarrow \overline{h}(\overline{z})$ we call H symmetric. Two basic examples of symmetric H are the space $\mathcal{P}$ of all polynomials and the space $\mathcal{F}(T)$

AMS (MOS) subject classifications (1970). Primary 41A30; Secondary 30A64.

[1] Research supported in part by NSF-MCS-74-07313-A02.