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Sheaf theory, by B. R. Tennison, London Mathematical Society Lecture Note Series, No. 20, Cambridge University Press, New York and London, 1976, vii + 164 pp., $8.95.

Perhaps the most pressing problem facing mathematics today is the increasing difficulty in communicating with nonmathematicians. The low percentage of new math Ph.D.'s with nonacademic jobs, the almost nonexistent intellectual interaction with other academic departments, and the increasingly common practise of having nonmathematicians teaching mathematics in their own disciplines illustrate this problem. In large measure it has been caused by an unhealthy overemphasis on abstraction during the past few decades. This particular book and, for that matter, all of the other books devoted solely to sheaf theory are prime examples of this overemphasis.

Since the mathematical style of graduate level texts is an important factor in determining the tastes of new mathematicians, these books and others which are written without reference to the concrete problems that gave rise to modern day mathematical edifices endanger the development of mathematics. The mathematical standards that are developed in our graduate students demand abstraction and elegant generalization while doing away with the necessity of justifying a result in terms of potential applications. This is a natural consequence of courses that rarely, if ever, present as a central topic a mathematical question of interest to a physicist or economist and then answer it in terms they could hope to understand. Instead a great deal of unnecessary generalization is introduced and stressed. As a result of this training in generalization, our new Ph.D.'s know how to check a theorem by determining its logical consequences or varying its hypotheses, but they rarely know how to apply the theorem to a problem of interest to a nonmathematician.

In the hands of an expert the power of abstraction and generalization is clear as Deligne's recent proof of the Ramanujan conjecture shows. Deligne was able to reduce this concrete conjecture about the partition function to the characteristic p Riemann hypothesis, and then by using the abstract, 'general nonsense' machinery of Grothendieck topologies and Grothendieck sheaf theory, he was able to prove the latter conjecture by an ingenious argument. Unfortunately in the hands of a novice mathematician, the power of abstraction and generalization too often leads to new "results" in abstract areas such as category theory, point set topology, or universal algebra while also giving him the impression of having done real mathematics. We badly need to correct this impression by emphasizing that the quality of a result is in large part determined by what it says about basic physical and mathematical problems.

Unfortunately the book under review will not, indeed, cannot, do this. It is a book devoted to a language, the language of sheaves, which may, by the end, leave the inexperienced reader with the feeling that he has been introduced to