BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 4, July 1977

HOLOMORPHIC CURVES IN ALGEBRAIC MANIFOLDS[1]

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A *holomorphic curve* in a complex manifold M is a nonconstant holomorphic map $f: \mathbf{C} \to M$. In 1927, R. Nevanlinna [**18**] created a new theory concerning the distribution of values of a holomorphic curve f in the complex projective line $\mathbf{CP}^1$. Nevanlinna's main result is that f assumes almost all values in $\mathbf{CP}^1$ "equally often," and those values that f fails to assume often enough have total "defect" at most 2. H. Cartan [**7**] generalized this "defect relation" to holomorphic curves f in $\mathbf{CP}^n$, counting how often f takes values in hyperplanes; L. Ahlfors [**2**] later reproved and extended Cartan's result, which he cast in a geometric form. Recently, J. Carlson and P. Griffiths [**6**] obtained new defect relations for holomorphic maps $f: \mathbf{C}^n \to M$, where M is an n-dimensional algebraic manifold, counting how often f takes values in divisors of a fixed holomorphic line bundle on M. Nevertheless, except for the results of Nevanlinna, Cartan and Ahlfors, very little is presently known about the distribution of values of holomorphic curves.

The first three sections describe the results of R. Nevanlinna [**18**], Cartan [**7**] and Ahlfors [**2**], and Carlson and Griffiths [**6**], respectively. (H. Cartan's brief, but not very well-known, proof of the defect relation for holomorphic curves in $\mathbf{CP}^n$ is given in §2.) §4 states some open problems on the distribution of values and existence of holomorphic curves in algebraic manifolds, as well as giving some recent results of R. Brody and of M. Green [**4**], [**5**], [**14**]. Finally, §5 gives a new proof of the Cartan-Ahlfors defect relation, which places it within the geometric framework of Carlson and Griffiths [**6**].

Other recent expositions of value distribution theory, which concentrate on mappings of several complex variables, are contained in the monograph by P. Griffiths [**27**] and the survey article by W. Stoll [**29**].

I learned about value distribution theory from Phillip Griffiths, Reese Harvey, and Yum-Tong Siu, whom I wish to thank for sharing their insights with me.

1. Nevanlinna theory. Let f be a nonconstant meromorphic function on the complex line $\mathbf{C}$. We regard f as a holomorphic curve in the complex projective line $\mathbf{CP}^1$ by identifying $\mathbf{CP}^1$ with the Riemann sphere $\mathbf{C} \cup \{\infty\}$. The earliest result on the values of f is *Liouville's Theorem* (1847) that f cannot omit an open set in $\mathbf{CP}^1$; i.e., $f(\mathbf{C})$ is dense in $\mathbf{CP}^1$. In 1879, Picard improved this

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This is an expanded version of an invited address given at the Cambridge, Massachusetts, meeting of the American Mathematical Society on October 25, 1975; received by the editors May 20, 1976.

AMS (MOS) subject classifications (1970). Primary 32-02, 32H99, 32H25, 30A70; Secondary 32H20.

[1]This work was partially supported by NSF Grant GP-40931 and by a Sloan Fellowship.