BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 4, July 1977

INFINITE LOOP SPACE THEORY

BY J. P. MAY[1]

Introduction. The notion of a generalized cohomology theory plays a central role in algebraic topology. Each such additive theory E^* can be represented by a spectrum E. Here E consists of based spaces E_i for $i \geqslant 0$ such that E_i is homeomorphic to the loop space ΩE_{i+1} of based maps $S^1 \to E_{i+1}$, and representability means that $E^n X = [X, E_n]$, the Abelian group of homotopy classes of based maps $X \to E_n$, for $n \geqslant 0$. The existence of the E_i for $i > 0$ implies the presence of considerable internal structure on E_0, the least of which is a structure of homotopy commutative H-space. Infinite loop space theory is concerned with the study of such internal structure on spaces.

This structure is of interest for several reasons. The homology of spaces so structured carries "homology operations" analogous to the Steenrod operations in the cohomology of general spaces. These operations are vital to the analysis of characteristic classes for spherical fibrations and for topological and PL bundles. More deeply, a space so structured determines a spectrum and thus a cohomology theory. In the applications, there is considerable interplay between descriptive analysis of the resulting new spectra and explicit calculations of homology groups.

The discussion so far concerns spaces with one structure. In practice, many of the most interesting applications depend on analysis of the interrelationship between two such structures on a space, one thought of as additive and the other as multiplicative.

The purpose of this talk is to give my view of the present state of infinite loop space theory, with emphasis on the intuitions behind the main concepts. There will be no formal statements, no proofs, very few complete definitions, and a general disregard of technical niceties. The details are now all written down, largely in [**20**], [**45**], and [**48**]. As these references indicate, this is not a historical survey, and I shall have little to say about the alternative theoretical approaches applicable to various portions of the material presented.[2] For the applications, it is sufficient, and necessary, to have a fully coherent framework, and my primary concern is to explain how the classifying spaces of geometric topology, the spectra of algebraic and topological K-theory, and Thom spectra appear and interact within one such framework and to show how the general abstract machinery is used to crank out explicit concrete calculations.

This is an expanded version of an invited address given at the annual meeting of the Society in San Antonio, Texas, on January 25, 1976; received by the editors July 2, 1976.

AMS (MOS) subject classifications (1970). Primary 55-02, 55B20, 55D35, 55F40; Secondary 55B15, 55D40, 55E50, 55F50, 18C15, 18D10, 18F25, 18H10.

[1]The author was partially supported by NSF grant GP-29075.

[2]Frank Adams is presently preparing a survey of this general area.