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RESIDUES AND CHARACTERISTIC CLASSES OF FOLIATIONS

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In this note we announce results and construct examples which show that a large number of characteristic classes for real foliations vary linearly independently. This generalizes the result of Thurston on the variation of the Godbillon-Vey invariant [T]. The method used is a special case of the general theory of residues of singular foliations due to Baum and Bott [BB].

DEFINITION. Let τ be a codimension q foliation on a manifold M. A vector field X on M is a Γ vector field for τ if $[X, Y]$ is tangent to τ whenever Y is tangent to τ. The singular set of X is the set of points where X is tangent to τ.

Let τ be an oriented codimension q foliation on an oriented manifold M. Let X be a Γ vector field for τ and assume the singular set of X consists of a single compact leaf N of τ. On $M - N$, τ and X span a foliation $\hat{\tau}$ of codimension $q - 1$. Let $\alpha^*: H^*(WO_{q-1}) \longrightarrow H^*(M - N; R)$ be the natural map associated to $\hat{\tau}$. Each element $\hat{\phi}$ of $H^{2q-1}(WO_{q-1})$ determines in a natural way an element ϕ of $H^{2q}(BU_q; R)$. Choose an embedded normal sphere bundle S of N in M and let $i: S \longrightarrow M - N$ be the inclusion. Denote by $\sigma: H^{2q-1}(S; R) \longrightarrow H^q(N; R)$ integration over the fiber of the sphere bundle S. On M, τ and X span a singular foliation with singular set N. Applying the theory of [BB], $\phi \in H^{2q}(BU_q; R)$, τ and X determine a cohomology class $\mathrm{Res}_\phi(\tau, X, N) \in H^q(N; R)$. We have

THEOREM 1. *For M, N, τ, and X as above and $\hat{\phi} \in H^{2q-1}(WO_{q-1})$,*

$$\sigma(i^*\alpha^*(\hat{\phi})) = \mathrm{Res}_\phi(\tau, X, N).$$

Let $\phi \in H^{2q}(BU_{q-1}; R)$. Then ϕ and $\hat{\tau}$ determine an element $S_\phi(\hat{\tau}) \in H^{2q-1}(S; R/Z)$, the Simons' character of $\hat{\tau}$, [ChS]. The element ϕ determines in a natural way an element ϕ in $H^{2q}(BU_q; R)$. We have

THEOREM 2. $S_\phi(\hat{\tau})[S] = \mathrm{Res}_\phi(\tau, X, N)[N] \bmod Z$, *where $[S]$ and $[N]$ are the homology classes determined by S and N.*

We give some examples which show that these residues are nontrivial and in fact vary linearly independently.

EXAMPLE 1. Denote by G the product of k copies of the special linear group SL_2R. Let K be a maximal compact subgroup of G and Γ a uniform dis-

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