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TOWARDS ALGEBRAIC COBORDISM

BY VICTOR SNAITH

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Abstract. A new description of cobordism is given and, by analogy, cobordism theories are defined for an arbitrary ring.

1. Let A be a ring with a unit. A cohomology theory, MA, might reasonably be called "the algebraic cobordism of A" if

(i) geometry over A gave rise to elements in $\pi_*(MA)$, and

(ii) the existence of Chern classes for A induced a transformation of cohomology theories from MA to the algebraic K-theory of A.

Below I sketch the construction of theories which often satisfy (i) and (ii). Details will appear in [2], [3].

Let X be a homotopy associative and commutative H-space. Let $T \subset \pi_*^S(X)$ be a finite subset of homogeneous elements. To this data is associated a periodic, commutative ring spectrum $X(T)$. $X(T)^*$ is the associated cohomology theory. For example, when $X = BU$ and T consists of the generator $B \in \pi_2(BU)$, then $X(T)_{2k} = \Sigma^2 BU$ and $\epsilon_{2k}\colon \Sigma^2 X(T)_{2k} \longrightarrow X(T)_{2k+2}$ is equal to

$$\Sigma^2(\Sigma^2 BU) \xrightarrow{h} \Sigma^2(S^2 \times BU) \xrightarrow{\Sigma^2(B \oplus \mathrm{id})} \Sigma^2(BU).$$

Here h is a Hopf construction and "id" is the identity map of BU.

When $X = BGLA^+$ for a ring A and $T \subset \pi_*^S(BGLA^+)$, $X(T)^*$ is called the *algebraic cobordism of A associated with T.* The terminology is motivated by (a)–(c) of the following result:

THEOREM 1.1. *Suppose* $\dim Y < \infty$; *then*:

(a) $BU(T)^0(Y) \simeq MU^{2*}(Y)$ *if* $T = \langle$*generator of* $\pi_2(BU)\rangle$;

(b) $BSp(T)^0(Y) \simeq MSp^{4*}(Y)$ *if* $T = \langle$*generator of* $\pi_4(BSp)\rangle$;

(c) $BO(T)^0(Y) \simeq MO^*(Y)$ *if* $T = \langle$*generator of* $\pi_1(BO)\rangle$;

(d) *if F is a finite field and T is a subset of $K_*(F)$ then* $BGLF^+(T)^0(Y) \simeq 0$;

(e) *if* $T = \langle$*generator of* $K_1(Z)\rangle$ *then* $BGLZ^+(T)^0(Y)$ *in general is a nontrivial group in which each element is of order* 2.

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