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FOURIER ANALYSIS ON COMPACT SYMMETRIC SPACE

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1. Let $L \supset K$ be Lie groups with complex Lie algebras $\mathfrak{l}_c$ and $\mathfrak{k}_c$. Assume $\mathfrak{k}_c$ has a linear complement $\mathfrak{b}$ in $\mathfrak{l}_c$ which is a subalgebra. For any σ in $\mathrm{LieHom}_{\mathbf{C}}(\mathfrak{b}, \mathbf{C})$ there is a unique germ of a C^ω function e^σ at $s_0 := K$ in $S := L/K$ such that $e^\sigma(s_0) = 1$ and $xe^\sigma = \sigma(x)e^\sigma$ (x in $\mathfrak{b}$). Now suppose S is connected, K is compact, and e^σ extends to an element of $C^\omega(S)$. Then (Harish-Chandra) $\varphi_\sigma(s) := \int_K e^\sigma(ks)\,dk$ is a spherical function in the sense that

$$\int_K \varphi_\sigma(gks)\,dk = \varphi_\sigma(gK)\varphi_\sigma(s).$$

For a Riemannian symmetric space of noncompact type Helgason [1], [2] extended Harish-Chandra's spherical transform theory to a Fourier theory in which functions of the form e^σ mimic the role of characters in classical Fourier theory on $\mathbf{R}^n$. Here we report that difficulties inherent in copying these ideas over to compact symmetric space have been overcome, at least for the rank one spaces.

2. Let $S := U/K$ be symmetric with U compact semisimple. Let G_c be a complexification of U and G a noncompact real form of G_c such that $K_0 := G \cap U$ is open in K, and maximal compact in G. Let $\mathfrak{g} = \mathfrak{k} + \mathfrak{a} + \mathfrak{n}$ be an Iwasawa decomposition and set $\mathfrak{b} := \mathbf{C}(\mathfrak{a} + \mathfrak{n})$. Then $\mathfrak{g}_c = \mathfrak{k}_c + \mathfrak{b}$ as in §1. Λ will denote the set of those λ in $\mathrm{LieHom}_{\mathbf{C}}(\mathfrak{b}, \mathbf{C})$ such that e^λ is in $C^\omega(S)$. $\Lambda|i\,\mathfrak{a}$ is the set of highest restricted weights of K-spherical representations of U. For λ in Λ let V_λ denote the corresponding irreducible U-submodule of $L^2(S)$. Then e^λ is the highest weight vector in V_λ. Define τ in $\mathrm{LieHom}_{\mathbf{C}}(\mathfrak{b}, \mathbf{C})$ by $\tau(x) := \mathrm{tr}(\mathrm{ad}\, x|\mathfrak{b})$ (x in $\mathfrak{b}$). Then τ is in Λ.

LEMMA 1. *There is a unique maximal connected, open, K-invariant neighborhood S_0 of s_0 in S on which $e^\tau \neq 0$. Then $e^\lambda \neq 0$ on S_0 for all λ in Λ.*

On S_0 define $e_*^\lambda := (e^{\lambda+\tau})^{-1}$. e_*^λ is the inverse transform kernel to e^λ. The aforementioned "inherent difficulty" of the subject is the singularity of e_*^λ off of S_0. Let $B := K/M$ where M is the centralizer of $\mathfrak{a}$ in K.

LEMMA 2. *For all uK in S_0, s in S, and λ in Λ*

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