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THE TRANSFER AND COMPACT LIE GROUPS

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1. **Introduction.** A map $\rho\colon X \longrightarrow Y$ between two spaces induces a homomorphism $\rho^*\colon h(Y) \longrightarrow h(X)$ between the cohomology groups of the spaces, where h is an arbitrary cohomology theory. In certain situations a transfer homomorphism $\tau^*\colon h(X) \longrightarrow h(Y)$ has been defined by Becker and Gottlieb, Dold and others. The compositions $\tau^* \circ \rho^*\colon h(Y) \longrightarrow h(Y)$ and $\rho^* \circ \tau^*\colon h(X) \longrightarrow h(X)$ are of considerable interest as they relate the cohomologies of X and Y. The first type of composition is relatively easy to compute. The second is, in general, quite difficult.

Let G be a compact Lie group with H and K arbitrary closed subgroups with associated l-universal classifying spaces BG, BH, BK. Let $\rho(H, G)\colon BH \longrightarrow BG$ be the natural projection. Then transfers $T(H, G)\colon h(BH) \longrightarrow h(BG)$, $T(K, G)\colon h(BK) \longrightarrow h(BG)$ are defined by Dold's definition where $T(H, G) = T_{\mathrm{id}}^{BH}$ in Dold's notation [D]. The main theorem is a double coset type theorem which generalizes the classical double coset theorem for finite groups [C–E, p. 257]. It is proved for arbitrary compact Lie groups.

2. **Main result.** Let $K|G|H$ be the double coset space obtained as the orbit space of the left action of K on G/H. This space breaks up into a finite disjoint union of orbit-type manifold components $\{M_i\}$. Let $g_i \in G$ be a representative of M_i. Let $\chi^{\#}(M_i) = \chi(\overline{M}_i) - \chi(\overline{M}_i - M_i)$ be the internal Euler characteristic of M_i. Then if $H^g = gHg^{-1}$ we have

THEOREM 1 (DOUBLE COSET).

$$\rho^*(K, G) \circ T(H, G) = \sum \chi^{\#}(M_i) T(H^{g_i} \cap K, K) \circ \rho^*(H^{g_i} \cap K, H^{g_i}) \circ Cg_i$$

where the sum is over the orbit-type manifold components of $K|G|H$. Cg: $h(BH) \longrightarrow h(BH^g)$ is the cohomology isomorphism induced by the obvious map from BH^g to BH.

This theorem holds where G is a compact Lie group and H and K are arbitrary closed subgroups.

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