main topics: *Local Observables and Constructive Field Theory.* Much thought has been given to the formulation of axioms in terms of local observables (e.g. a lattice of algebras of bounded operators) whose physical interpretation includes all possible experimental measurements which could be carried out in a space (or space-time) region. This section covers much early work of Haag, Kastler and Araki, but covers only a small part of the interesting results on superselection rules established by Haag, Doplicher and Roberts. The second aspect of Part 6 is an introduction to the existence problem for fields satisfying the axioms and to the analysis of detailed properties of solutions to model equations. This "constructive field theory" has been another major focus in the study of quantum fields over the past ten years. The authors devote the final section of their book to a brief but comprehensive survey of this work up to 1971, when their manuscript was completed.

All in all, the book provides a readable introduction to a large area of mathematical physics. In trying to include many things, the authors are occasionally incomplete or sloppy in minor ways. However, the book complements well the older books on axiomatic field theory by Jost and by Streater and Wightman, and a recent review by Streater in *Reports of progress in physics.* A mathematician interested in physics must be willing to learn some of the language and definitions of the physicist. This book is a good place to begin.

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Proof theory, by Gaisi Takeuti, Studies in Logic and the Foundations of Mathematics, vol. 81, North-Holland/American Elsevier, Amsterdam, Oxford, New York, 1975, vii + 372 pp., $35.50.

Takeuti places himself squarely in the line of development of Hilbert and Gentzen, which we begin by retracing. Proof theory was conceived by Hilbert as the means to carry out his grand program to secure the foundations of mathematics by purely mathematical means which were to be of the most elementary and evident kind. The logical structure of mathematical practice had been successfully mirrored within a variety of formal systems S for algebra, geometry, number theory, analysis, and set theory, all logically based in the predicate calculus (the logic of propositional connectives and quantifiers). The consistency of each such S is a combinatorial proposition which may be shown to be equivalent to a number-theoretical statement of the form (for all natural numbers n) $f(n) = g(n)$, where f, g are effectively computable. Hilbert expected to be able to derive such statements using only quantifier-free logic, recursive definitions of functions and proof by induction. Each derivation of a specific numerical statement by these means has a finite model. This differs from the situation where quantifiers are essentially involved in the reasoning; thus even where the variables of S range over natural numbers one may say that application of the reasoning of the predicate calculus in S implicitly involves infinitistic concepts.

To elaborate a bit: Hilbert spoke of the quantifier-free numerical statements as the "real" ones, and of statements involving noncombinatorial concepts or