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THE HASSE NORM PRINCIPLE FOR ABELIAN EXTENSIONS OF NUMBER FIELDS

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1. Introduction. Let K be a finite extension of $\mathbf{Q}$, the field of rational numbers, and let L be a finite abelian extension of K. We say that the Hasse norm principle is valid for L/K if the following statement is true: each nonzero element of K is the norm of an element of L if and only if it is the norm of an element from each completion of L. It is well known that the Hasse norm principle is valid when L/K is cyclic, but the Hasse norm principle is not always valid for L/K when L/K is not cyclic (see [1, p. 199]). Our goal in this paper is to give an explicit, computable algorithm for determining whether the Hasse norm principle is valid for a given finite abelian extension L/K. Proofs will appear elsewhere. Before stating our results, we remark that Garbanati (see [2]) has obtained such an algorithm for certain finite abelian extensions L of $\mathbf{Q}$ of prime power degree, and Razar (see [3]) has also obtained some interesting results on the Hasse norm principle. Razar's results include results equivalent to Theorems 1 and 2 in the next section.

2. Main results.

THEOREM 1. *Let K be a finite extension of* $\mathbf{Q}'$, *and let L be a finite abelian extension of K. Let $l_1, \ldots, l_t$ be the distinct prime numbers dividing the order of* $\mathrm{Gal}(L/K)$, *and let L_i be the maximal l_i-extension of K contained in L, $1 \leqslant i \leqslant t$. Then the Hasse norm principle is valid for L/K if and only if the Hasse norm principle is valid for each L_i/K, $1 \leqslant i \leqslant t$.*

REMARK. Theorem 1 reduces the problem to the case where L/K is a finite abelian l-extension, where l is a prime number.

THEOREM 2. *Let K be a finite extension of* $\mathbf{Q}$, *and let L be a finite abelian l-extension of K, where l is a prime number. Let M be the maximal extension of K of exponent l contained in L. Then the Hasse norm principle is valid for L/K if and only if the Hasse norm principle is valid for M/K.*

REMARK. Theorem 2 reduces the problem to the case where L/K is a finite abelian l-extension with exponent l.

THEOREM 3. *Let K be a finite extension of* $\mathbf{Q}$, *and let L be a finite abelian*

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