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DISCONTINUOUS HOMOMORPHISMS FROM $C(X)$

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Let X be an infinite compact Hausdorff space, let $C(X)$ denote the algebra of continuous complex-valued functions on X, and let $|\cdot|_X$ be the uniform norm on X. We announce two completely independent proofs of the following theorem.

THEOREM. *Assuming the continuum hypothesis* (*CH*), *there exists a discontinuous monomorphism from* $(C(X), |\cdot|_X)$ *into certain Banach algebras, and there is an incomplete algebra norm on* $C(X)$.

The conjecture that every algebra norm $\|\cdot\|$ on $C(X)$ is equivalent to the uniform norm arises naturally from a theorem of Kaplansky in 1949 that necessarily $\|f\| \geqslant |f|_X$ $(f \in C(X))$: see [9, 10.1]. The seminal paper on the automatic continuity of homomorphisms from $C(X)$ is the 1960 paper of Bade and Curtis [2] in which, for example, it is proved that there is a discontinuous homomorphism from $C(X)$ if and only if there is a radical homomorphism, a nonzero homomorphism from a maximal ideal of $C(X)$ into a commutative radical Banach algebra. In 1967, it was proved by Johnson that every homomorphism from certain noncommutative C^*-algebras is continuous: see [9, 12.4]. Sinclair proved recently that the existence of a discontinuous homomorphism is equivalent to the existence of an algebra norm on $C(X)/I$ for some nonmaximal prime ideal I of $C(X)$ [9, 11.7], and this was proved independently in [4]. It follows from the work of each of the present authors that such a norm exists provided $|C(X)/I| = \aleph_1$. Assuming CH, such an ideal exists for each X, and every nonmaximal prime ideal has this property if X is separable, but, if X is not separable, there may exist a prime ideal J such that $C(X)/J$ is not normable [4].

The work of the first author is contained in [3]. Write C for the continuous real-valued functions on $\beta\mathbf{N}$, let $\mathfrak{p} \in \beta\mathbf{N}\backslash\mathbf{N}$, let $M_\mathfrak{p} = \{f \in C: f(\mathfrak{p}) = 0\}$, let $J_\mathfrak{p} = \{f = 0 \text{ near } \mathfrak{p}\}$, and let $A = M_\mathfrak{p}/J_\mathfrak{p}$. Then the quotient field of A is a real-closed totally ordered η_1-field of cardinality $2^{\aleph_0}$, and hence is a nonstandard model of the reals. For $\sigma \geqslant 1$, let $\Omega_\sigma = \{\operatorname{Re} z > 1, |z| < \sigma\}$, let $\mathsf{A}_\sigma = C^*(\overline{\Omega}_\sigma) \cap \mathcal{O}(\Omega_\sigma)$, and let $\mathsf{A}_\infty = \operatorname{ind}\lim \mathsf{A}_\sigma$: A_∞ is an algebra of germs of analytic functions on 'half-neighborhoods' of infinity. The major part of [3] is the construction of a momorphism $A \longrightarrow \mathsf{A}_\infty$. At one point, a recent result of

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