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Representations of commutative semitopological semigroups, by Charles F. Dunkl and Donald E. Ramirez, Lecture Notes in Mathematics, no. 435, Springer-Verlag, Berlin, Heidelberg, New York, 1975, 181 + vi pp., $8.60.

It is probably unnecessary to say that a *semigroup* is a set with an associative multiplication; yet it may be useful to state in the beginning that a *topological semigroup* is a semigroup equipped with a topology making multiplication jointly continuous, and that a *semitopological semigroup* differs from it insofar as the multiplication may be only separately continuous in each variable. A reader who is somewhat familiar with topological groups but less acquainted with semigroups may wonder about the necessity of this distinction; it seems to play a small role in group theory. The reason is that, for the most commonly treated types of groups such as those on locally compact or Polish (completely metrizable 2nd countable spaces), separate continuity of multiplication implies the axioms of a topological group (Ellis, Effros); thus the distinction is largely unnecessary for groups. For semigroups, however, the only class other than groups showing a similar behavior is that of *semilattices*, i.e. commutative semigroups in which all elements are idempotent; J. D. Lawson recently showed that every compact semitopological semilattice must be topological. (In this general context, he proved that every subgroup of a compact semitopological semigroup is in fact a topological group; this fails for a subsemilattice.) But in general, separate continuity on semigroups does not imply joint continuity. Moreover, semitopological semigroups which are not topological arise quite naturally in great variety, notably in analysis. Indeed in many instances semigroups occur here as semigroups of bounded operators on a Banach space; only if one considers the operator norm topology will such semigroups be automatically topological; in the more commonly considered operator topologies such as the strong or weak (or, in the case of Hilbert spaces, ultraweak) topology they will be semitopological, but rarely topological. It is therefore only natural that the functional analysts, notably when they prepare to talk on representation theory, should immediately turn to semitopological semigroups, as do Dunkl and Ramirez as soon as they give us the title of their book.

For numerous reasons the study of topological and semitopological semigroups has a different flavor from the investigation of topological groups. It is much more recent and much less developed than the latter, despite the existence of considerable journal literature on the subject spread over the last quarter century. Even when we compare compact groups and semigroups, the most striking difference is the absence of invariant integration on semigroups; some compact semigroups have no invariant measure (on either side); and in fact no compact semigroup which is not a group has an invariant measure whose support is the whole semigroup. Consequently, L^2-representations of semigroups are rare and not as organic as in the case of groups. Other phenomena compound this difficulty and make finite dimensional linear