to introduce the states also, and develop the theory along lines in which both the logic and the space of its states play equally fundamental roles. The phenomenon of complementarity and the problems connected with the existence of a lattice structure on a logic then appear to emerge more clearly out of the manner in which the observables and states are interconnected. The expositions of Mackey and Zierler are of this type. To dismiss one of the crucial aspects of the subject in such a perfunctory manner as Piron has done is, at the least, very misleading. I would also like to point out that Piron makes no reference to the work of Zierler on the characterization of standard logics, although Zierler's work was done more or less simultaneously with Piron's and independently of it. There are many such instances of a lack of proper care in giving references to the work of others scattered throughout this book, making this exposition somewhat distorted. The reader who wants to be informed in depth on the various aspects of the subject and the extensive literature on these questions would do well to consult the volume entitled *The logico-algebraic approach to quantum mechanics*, edited by C. A. Hooker (D. Reidel Publishing Company).

V. S. VARADARAJAN

BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 2, March 1977

Non-archimedean fields and asymptotic expansions, by A. H. Lightstone and A. Robinson, North-Holland Mathematical Library, vol. 13, North-Holland/American Elsevier, Amsterdam and New York, 1975, 204 + x pp., $24.95.

The aim of this short book is to show that nonarchimedean fields and nonstandard analysis form an excellent setting for the study of asymptotic expansions. The authors have been quite successful in achieving this goal. One wishes there were more, but the terminal illness of Abraham Robinson, who wrote the first draft, prevented further collaboration on Harald Lightstone's final manuscript. Since both authors are now deceased, it will be up to others to further their ideas.

An asymptotic expansion for a function f with respect to an "asymptotic sequence" of functions ϕ_i is a formal series $\sum_{i=0}^{\infty} a_i\phi_i$ such that while the sequence of partial sums $S_n(x) = \sum_{i=0}^{n} a_i\phi_i(x)$ may diverge at a given x, there may yet exist an n_x (in practice, small) such that $S_{n_x}(x)$ is a satisfactory approximation to $f(x)$. For the most part, the book deals with real, rather than complex, valued functions. A sequence of real-valued functions $\phi_0, \phi_1, \ldots$ is called an asymptotic sequence if there is a neighborhood of $+\infty$ in the real line R on which each ϕ_i is defined and nonvanishing and for each n in the natural numbers $N = \{0, 1, 2, \ldots\}$ we have $\phi_{n+1} = o(\phi_n)$, i.e., $\lim_{x\to\infty} \phi_{n+1}(x)/\phi_n(x) = 0$. Given an asymptotic sequence $\{\phi_i\}$, a sequence of real numbers $\{a_i\}$, and a real-valued function f defined on some neighborhood $(t, +\infty)$ of $+\infty$ in R, the formal sum $\sum a_i\phi_i$ is called an asymptotic expansion for f, and we write $f \sim \sum a_i\phi_i$, if for each $n \in N$, $f - \sum_0^n a_i\phi_i = o(\phi_n)$. One may think of the nth error as being a higher order of infinitesimal than the last term adjoined to the series.