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Univalent functions–selected topics, by Glenn Schober, Lecture Notes in Mathematics, no. 478, Springer-Verlag, Berlin, Heidelberg, New York, 1975, iv + 200 pp.

The term univalent applied to a mapping means simply that it is one-to-one. However the combination "univalent functions" has a much more specific meaning, referring to regular (holomorphic) or meromorphic functions which determine one-to-one mappings. They may be considered in various domains of definition, even on a Riemann surface, but attention is often directed to certain specific classes; two of the most important are denoted by S and Σ. The first consists of functions $f(z)$ regular and univalent for $|z| < 1$ with Taylor expansion about the origin

$$z + \sum_{n=2}^{\infty} A_n z^n. \tag{1}$$

The second consists of functions $f(z)$ meromorphic and univalent for $|z| > 1$ with Laurent expansion about the point at infinity

$$z + \sum_{n=0}^{\infty} c_n z^{-n}. \tag{2}$$

The theory of univalent functions had its beginnings in results of Koebe obtained in 1907 and 1909 which may be stated as follows.

I. *There exists an absolute constant κ such that for $f \in S$ the function values $w = f(z)$ for $|z| < 1$ fill the circle $|w| < \kappa$ where κ is the largest value for which this is true.*

II. *There exist positive quantities $m_1(r)$, $M_1(r)$ depending only on r such that for $f \in S$, $|z| = r$,*

$$m_1(r) \leqslant |f(z)| \leqslant M_1(r).$$

III. *There exist positive quantities $m_2(r)$, $M_2(r)$ depending only on r such that for $f \in S$, $|z| = r$,*

$$m_2(r) \leqslant |f'(z)| \leqslant M_2(r).$$

These results aroused great interest and a number of people began to work in the field. In particular, the first consistent method was introduced, the area method, and used by Gronwall, Bieberbach and Faber. This method utilizes the simple fact that the area enclosed by the image under $f \in \Sigma$ of the circle $|z| = r$ $(r > 1)$ is positive. Expressing this in terms of the coefficients of f it is easy to obtain the Area Theorem:

$$\sum_{n=1}^{\infty} n|c_n|^2 \leqslant 1.$$

From this one can easily prove that $\kappa = \frac{1}{4}$ in I and obtain the precise expressions for $m_1(r)$, $M_1(r)$, $m_2(r)$, $M_2(r)$ in II and III. Further, Bieberbach