

because calculus is about the real numbers. The book offers no evidence that the hyperreal numbers are anything except a device for proving theorems about the real numbers. They are not even an efficient device, depending as they do on axioms V^* and VI^* , among other things.

The technical complications introduced by Keisler's approach are of minor importance. The real damage lies in his obfuscation and devitalization of those wonderful ideas. No invocation of Newton and Leibniz is going to justify developing calculus using axioms V^* and VI^* —on the grounds that the usual definition of a limit is too complicated!

Although it seems to be futile, I always tell my calculus students that mathematics is not esoteric: It is common sense. (Even the notorious ϵ, δ definition of limit is common sense, and moreover is central to the important practical problems of approximation and estimation.) They do not believe me. In fact the idea makes them uncomfortable because it contradicts their previous experience. Now we have a calculus text that can be used to confirm their experience of mathematics as an esoteric and meaningless exercise in technique.

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Basic linear partial differential equations, by François Trèves, Academic Press, New York, 1975, xvii + 470 pp., \$29.50.

How, and why, would one write 470 pages on “basic” linear PDE, a subject which advanced calculus texts purport to treat in 50 or 60 pages? It is not because Trèves has enlarged the stock of basic equations: the standard problems and their immediate generalizations essentially fill the book. It is not because of space spent on preliminaries: distribution theory and basic functional analysis are assumed. The answer may be found by considering another question: How does one approach a typical basic problem in a modern way?

Consider a simple “mixed initial-boundary value problem” for the heat equation. The object is, given a function $u_0(x)$, $x \in [-1, 1]$, to find a function u defined on $[-1, 1] \times [0, \infty)$ such that

$$(1) \quad \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad u(x, 0) = u_0(x), \quad u(\pm 1, t) \equiv 0.$$

Let us look at (1) as an ordinary differential equation for a vector-valued function. We denote by A the linear operator $(d/dx)^2$, with domain a suitable space of functions on $[-1, 1]$ which vanish at the endpoints. We let X be a space of functions containing the domain of A and the initial value u_0 , and look for $u: [0, \infty) \rightarrow X$ such that

$$(2) \quad \frac{du}{dt} = Au, \quad u(0) = u_0.$$