BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 83, Number 1, January 1977

THE CONFORMAL STRUCTURE AND GEOMETRY OF TRIPLY PERIODIC MINIMAL SURFACES IN R^3

BY WILLIAM H. MEEKS, III[1]

Communicated by M. A. Rosenlicht, September 1, 1976

ABSTRACT. Working within the conformal category, we develop complementary existence and rigidity theories for periodic minimal surfaces in $\mathbf{R}^n$.

We will call a compact Riemann surface *M periodic* if it conformally minimally immerses in a flat three-torus T^3. By lifting to the universal cover of T^3, these periodic surfaces become the proper triply periodic minimal surfaces in $\mathbf{R}^3$.

We find that the compactness of a minimal surface M in T^3 gives rise to restrictions on the conformal type of M. Frequently, these conformal restrictions give nontrivial geometric information about the lifted minimal surface in $\mathbf{R}^3$. For this reason, we consider the following fundamental problems:

(1) Which compact Riemann surfaces are periodic?

(2) How does the conformal structure of a periodic surface influence its geometry?

Our first result on these questions is that a surface of genus two is never periodic. Since every surface of genus two is hyperelliptic, this follows from our more general result that a hyperelliptic Riemann surface of even genus is never periodic. We also find another family of nonperiodic surfaces: Any nonsingular curve of degree four in CP^2 fails to be periodic. Thus, the classical Fermat curve of degree four in CP^2 given in homogeneous coordinates by $x^4 + y^4 + z^4 = 0$ provides a good example of a nonperiodic surface. The techniques of proof used here consist of a study of the Gauss map of a minimal surface and the canonical curve of a Riemann surface.

Besides finding conformal obstructions to periodicity, we also begin the development of a general existence theory. Much of this existence theory is based on our rigidity theorems for periodic and nonperiodic minimal surfaces in $\mathbf{R}^3$ and on the study of the canonical curve of a Riemann surface. One consequence of joining these theories is that we can show the Schwartz diamond surface can be joined to its conjugate surface through minimal surfaces in flat three-tori.

The following is a list of our basic results.

THEOREM 1. *There exists a real* 5*-dimensional family V of periodic hyperelliptic surfaces of genus* 3. *The surfaces in V are the two-sheeted covers of S^2 branched over* 8 *antipodal points.*

AMS (MOS) subject classifications (1970). Primary 53A10.

[1]The author was supported by NSF grant MPS71-02597.